BUSINESS STATISTICS COMM 215
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Lesson 8: Chapter 9 Hypothesis Testing

PROBLEM # 8.1

The null hypothesis is a statement about the value of a population parameter. It is assumed to be true unless we have evidence to the contrary. The alternative hypothesis is an assertion that holds if the null hypothesis is false. The null hypothesis is not always the same as the verbal claim or assertion that led to the test. The null hypothesis must always contain the equal sign. If the directional claim does not contain an equal sign, then the claim is put in the alternative hypothesis and the opposite is put in the null hypothesis.

PROBLEM # 8.4

The test would be one-tail and the appropriate null and alternative hypotheses would be H_0: $\pi \geq 0.85$ and H_1: $\pi < 0.85$.

PROBLEM # 8.6

The decision rule specifies the conclusion to be reached for a given outcome of the test (e.g., Reject H_0 if the calculated $z > 1.96$). The decision rule helps us to decide whether to reject H_0 or fail to reject H_0 for a hypothesis test.

PROBLEM # 8.7

If the sample size is large ($n \geq 30$), the central limit theorem assures us that the distribution of sample means will be approximately normally distributed regardless of the shape of the underlying population. The larger the sample size, the better this approximation becomes. When the central limit theorem applies, we may use the standard normal distribution to identify the critical values for the test statistic when σ is known.

PROBLEM # 8.8

If $n < 30$, we must assume that the underlying population is normally distributed in order to use the z-statistic.

PROBLEM # 8.10

Right-tail test and z=1.54: p-value = $P(z \geq 1.54) = 1.0000 - 0.9382 = 0.0618$

Left-tail test and z=-1.03: p-value = $P(z \leq -1.03) = 0.1515$

Two-tail test and z=1.27: p-value = $2P(z \leq -1.83) = 2(0.0336) = 0.0672$

PROBLEM # 8.12

Null and alternative hypotheses:

H_0: $\mu \geq 5.00$ (no decline in spending) H_1: $\mu < 5.00$ (a decline)

Level of significance: $\alpha = 0.05$

Test results: $\bar{x} = 4.20$, n = 18 (known: $\sigma = 1.80$ and the population is normally distributed.)

Calculated value of test statistic: $$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{4.20 - 5.00}{1.80/\sqrt{18}} = -1.89$$

Critical value: $z = -1.645$ (in the normal distribution the area to the left of $z = -1.645$ is 0.05).

Decision rule: Reject H_0 if the calculated $z < -1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0.

Decision: At the 0.05 level, the results suggest a decline in spending on popcorn and snacks at the cinema complex. It appears the average amount spent is now less than $5.00.

Using the standard normal distribution table, we can find the approximate p-value as the area to the left of z = -1.89. This is 0.0294.

Given the summary data, we can also carry out this z-test using the Test Statistics workbook that accompanies Data Analysis Plus. For this left-tail test, the p-value (0.0297) is less than the 0.05 level of significance being used to reach a conclusion, so the null hypothesis is rejected. For a true null hypothesis, there is only a 0.0297 probability that a sample mean this much less than $5.00 would occur by chance.

	A	B	C	D
1	**z-Test of a Mean**			
2				
3	**Sample mean**	4.20	**z Stat**	**-1.89**
4	**Population standard deviation**	1.80	**P(Z<=z) one-tail**	**0.0297**
5	**Sample size**	18	**z Critical one-tail**	**1.645**
6	**Hypothesized mean**	5.00	**P(Z<=z) two-tail**	**0.0593**
7	**Alpha**	0.05	**z Critical two-tail**	**1.960**

PROBLEM # 8.14

The t statistic should be used in carrying out a hypothesis test for the mean when σ is unknown. When $n < 30$, we must assume the population is approximately normally distributed.

PROBLEM # 8.15

Null and alternative hypotheses: $H_0: \mu = 24.0$ $H_1: \mu \neq 24.0$

Level of significance: $\alpha = 0.01$

Test results: $\bar{x} = 25.9$, $s = 4.2$, $n = 40$

Calculated value of test statistic: $$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{25.9 - 24.0}{4.2/\sqrt{40}} = 2.861$$

Critical values: t = -2.708 and t = 2.708

For this test, $\alpha = 0.01$ and d.f. = (n - 1) = (40 - 1) = 39.

Referring to the 0.01/2 = 0.005 column and the 39th row of the t table, the critical values are t = -2.708 and t = 2.708.

Decision rule: Reject H_0 if the calculated t < -2.708 or > 2.708, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0.

Decision: At the 0.01 level, the results suggest that the population mean is not equal to 24.0.

PROBLEM # 8.18

The normal distribution is a good approximation for the binomial distribution when $n\pi$ and $n(1-\pi)$ are both ≥ 5.

PROBLEM # 8.20

Null and alternative hypotheses: $H_0: \pi \geq 0.50$ $H_1: \pi < 0.50$

Level of significance: $\alpha = 0.05$

Test results: p = 0.47, n = 1000

Calculated value of test statistic: $$z = \frac{p - \pi_0}{\sigma_p} = \frac{0.47 - 0.50}{\sqrt{0.5(1-0.5)/1000}} = -1.90$$

Critical value: z = -1.645

Decision rule: Reject H_0 if the calculated z < -1.645, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0.

Decision: At the 0.05 level, the results suggest that the population proportion is less than 0.50.

Given the summary data, we can also use the Test Statistics workbook that accompanies

Data Analysis Plus. For this left-tail test, the p-value (0.029) is less than the 0.05 level of significance being used to reach a conclusion, so reject the null hypothesis.

	A	B	C	D
1	**z-Test of a Proportion**			
2				
3	**Sample proportion**	0.47	**z Stat**	**-1.90**
4	**Sample size**	1000	**P(Z<=z) one-tail**	**0.029**
5	**Hypothesized proportion**	0.50	**z Critical one-tail**	**1.645**
6	**Alpha**	0.05	**P(Z<=z) two-tail**	**0.058**
7			**z Critical two-tail**	**1.960**

PROBLEM # 8.21

Null and alternative hypotheses:

H_0: $\pi \leq 0.02$ (the proportion of defectives is no more than 0.02)

H_1: $\pi > 0.02$ (the proportion of defectives is greater than 0.02)

Level of significance: We will use $\alpha = 0.05$ in carrying out this right-tail test.

Test results: $p = 0.04$, $n = 300$

Calculated value of test statistic: $$z = \frac{p - \pi_0}{\sigma_p} = \frac{0.04 - 0.02}{\sqrt{0.02(1-0.02)/300}} = 2.47$$

Critical value: $z = 1.645$

Decision rule: Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Conclusion: Calculated test statistic falls in rejection region, reject H_0.

Decision: At the 0.05 level, the results suggest that the supplier's claim is not correct. The true percentage of defectives in the shipment appears to be greater than 2%.

Given the summary data, we can also use the Test Statistics workbook that accompanies
Data Analysis Plus. For this right-tail test, the p-value (0.007) is less than the 0.05 level of significance being used to reach a conclusion, so reject the null hypothesis.

	A	B	C	D
1	**z-Test of a Proportion**			
2				
3	**Sample proportion**	0.04	**z Stat**	**2.47**
4	**Sample size**	300	**P(Z<=z) one-tail**	**0.007**
5	**Hypothesized proportion**	0.02	**z Critical one-tail**	**1.645**
6	**Alpha**	0.05	**P(Z<=z) two-tail**	**0.013**
7			**z Critical two-tail**	**1.960**

PROBLEM # 8.22

Null and alternative hypotheses:

H_0: $\pi = 0.65$ (percentage who prefer electric heating has not changed)

H_1: $\pi \neq 0.65$ (percentage who prefer electric heating has changed)

Level of significance: $\alpha = 0.05$

Test results: $p = 0.60$, $n = 200$

Calculated value of test statistic: $$z = \frac{p - \pi_0}{\sigma_p} = \frac{0.60 - 0.65}{\sqrt{0.65(1-0.65)/200}} = -1.48$$

Critical values: $z = -1.96$ and $z = 1.96$

Decision rule: Reject H_0 if the calculated $z < -1.96$ or > 1.96, otherwise do not reject.

Conclusion: Calculated test statistic falls in nonrejection region, do not reject H_0.

Decision: At the 0.05 level, we cannot conclude that the percentage of residential energy consumers who prefer to heat with electricity instead of gas has changed from

65%. The difference between the hypothesized population proportion and the sample proportion is judged to have been merely the result of chance variation.

The p-value for this two-tail test is twice the area to the left of z = -1.48, or 2(0.0694) = 0.1388.

Because the p-value is not less than 0.05, we do not reject the null hypothesis. Using the Test Statistics workbook, the corresponding results are shown below.

	A	B	C	D
1	**z-Test of a Proportion**			
2				
3	**Sample proportion**	0.60	**z Stat**	**-1.48**
4	**Sample size**	200	**P(Z<=z) one-tail**	**0.069**
5	**Hypothesized proportion**	0.65	**z Critical one-tail**	**1.645**
6	**Alpha**	0.05	**P(Z<=z) two-tail**	**0.138**
7			**z Critical two-tail**	**1.960**

PROBLEM # 8.24-

A p-value is the exact level of significance associated with the calculated value of the test statistic. It is the most extreme critical value that the test statistic would be capable of exceeding. If p-value < α, reject H_0 and if p-value ≥ α, do not reject H_0.

PROBLEM # 8.25-

Since p-value = 0.03 is less than α = 0.05, the null hypothesis would be rejected. The sample result is more extreme than you would have been willing to attribute to chance.

PROBLEM # 8.26-

If we are unable to reject H_0, then the p-value is not less than the level of significance being used (α = 0.01), or p-value ≥ 0.01.