Lesson 6 –Sampling Distributon

At Home Problem Solutions

PROBLEM # 6.7

a) $P(\bar{X} > 1{,}050)$

$P(\bar{X} > 1050) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1050-1000}{200/\sqrt{16}}\right)$ *= P(Z > 1.00) = 1 – P(Z < 1.00) = 1 – .8413 =.1587*

b) $P(\bar{X} < 960)$

$P(\bar{X} < 960) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{960-1000}{200/\sqrt{16}}\right)$ *= P(Z < –.80) = .2119*

c) $P(\bar{X} > 1{,}100)$

$P(\bar{X} > 1100) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1100-1000}{200/\sqrt{16}}\right)$ *= P(Z > 2.00) = 1 – P(Z < 2.00) = 1 – .9772 =.0228*

PROBLEM # 6.9

Given that μ = 80, σ = 20, n = 100, then $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2.0$

$P(\bar{x} > 8300/100) = P(\bar{x} > 83 \text{ pounds}) = P(z > \frac{\bar{x}-\mu}{\sigma_{\bar{x}}}) = P(z > \frac{83-80}{2})$

$= P(z > 1.5) = 1.0000 - 0.9332 = 0.0668$

Thus, 6.68% of the flights will require an extra 500 pounds of fuel.

PROBLEM # 6.10

The professor needs to know the mean and standard deviation of the population of the weights of elevator users and that the distribution is not extremely nonnormal.

PROBLEM # 6.11 Refer to PROBLEM # 6.10.

$P(\bar{X} > 1{,}140/16) = P(\bar{X} > 71.25) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{71.25-75}{10/\sqrt{16}}\right)$ *= P(Z > –1.50)*

= 1 – P(Z < –1.50) = 1 – 0668 = .9332

PROBLEM # 6.12

P(Total time > 300) = $P(\bar{X} > 300/60) = P(\bar{X} > 5) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{5-4.8}{1.3/\sqrt{60}}\right)$ *= P(Z > 1.19)*

= 1 – P(Z < 1.19) = 1 – .8830 = .1170

PROBLEM # 6.14

a.

$$P(\hat{P} < .22) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.22-.25}{\sqrt{(.25)(1-.25)/500}}\right) = P(Z < -1.55) = .0606$$

b. Repeat Part a with n=800.

$$P(\hat{P} < .22) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.22-.25}{\sqrt{(.25)(1-.25)/800}}\right) = P(Z < -1.96) = .0250$$

PROBLEM # 6.15

a. What is the probability that a randomly selected woman is taller than 66 inches?

$$P(X > 66) = P\left(\frac{X-\mu}{\sigma} > \frac{66-64}{2}\right) = P(Z > 1.00) = 1 - P(Z < 1.00) = 1 - .8413 = .1587$$

b. A random sample of four women is selected. What is the probability that the sample mean height is greater than 66 inches?

$$P(\overline{X} > 66) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{66-64}{2/\sqrt{4}}\right) = P(Z > 2.00) = 1 - P(Z < 2.00) = 1 - .9772 = .0228$$

c. What is the probability that the mean height of a random sample of 100 women is great than 66 inches?

$$P(\overline{X} > 66) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{66-64}{2/\sqrt{100}}\right) = P(Z > 10.00) = 0$$

PROBLEM # 6.16

a. What proportion of the class has a midterm mark of less than 75?

$$P(X < 75) = P\left(\frac{X-\mu}{\sigma} < \frac{75-78}{6}\right) = P(Z < -.50) = .3085$$

b. What is the probability that a class of 50 has an average midterm mark that is less than 75?

$$P(\overline{X} < 75) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{75-78}{6/\sqrt{50}}\right) = P(Z < -3.54) = 1 - P(Z < 3.54) = 1 - 1 = 0$$

PROBLEM # 6.19

$$P(\hat{P} < .75) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.75-.80}{\sqrt{(.80)(1-.80)/100}}\right) = P(Z < -1.25) = .1056$$

PROBLEM # 6.20

$P(\hat{P} > .04) = P\left(\frac{\hat{P} - p}{\sqrt{p(1-p)/n}} > \frac{.04 - .02}{\sqrt{(.02)(1-.02)/800}}\right) = P(Z > 4.04) = 1 - P(Z < 4.04) = 1 - 1 = 0;$

The defective rate appears to be larger than 2%.

PROBLEM # 6.21

a. If this claim is true what is the probability that in a random sample of 600 customers less than 45% are satisfied?

$P(\hat{P} < .45) = P\left(\frac{\hat{P} - p}{\sqrt{p(1-p)/n}} < \frac{.45 - .50}{\sqrt{(.50)(1-.50)/600}}\right) = P(Z < -2.45) = .0071$

b. Suppose that in a random sample of 600 customers, 270 express satisfaction with the bookstore. What does this tell you about the bookstore's claim?

The claim appears to be false.