Lesson 3 At Home Problem Solutions

Problem # 3.3 Computer the number of ways you can select n elements from N elements for each of the following:

a. n=2, N=5 $\binom{N}{n} = \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5*4*3*2*1}{2*1*3*2*1} = \frac{120}{12} = 10$

b. n=3,N=6 $\binom{N}{n} = \binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6*5*4*3*2*1}{3*2*1*3*2*1} = \frac{720}{36} = 20$

c. n=5,N=20 $\binom{N}{n} = \binom{20}{5} = \frac{20!}{5!(20-5)!} = \frac{20*19*18*....3*2*1}{5*4*3*2*1*15*14*13...3*2*1} = \frac{2.4329*10^{18}}{1.5692*10^{14}} = 15{,}504$

Problem # 3.6

Determine the probabilities of the following events.

a. Adams loses.
   P(Adams loses) = P(Brown wins) + P(Collins wins) + P(Dalton wins) = .09 + .27 + .22 = .58
b. Either Brown or Dalton wins.
   P(either Brown or Dalton wins) = P(Brown wins) + P(Dalton wins) = .09 + .22 = .31
c. Adams, Brown, or Collins wins
   P(either Adams, Brown, or Collins wins) = P(Adams wins) + P(Brown wins) + P(Collins wins)
   = .42 + .09 + .27 = .78

Problem # 3.9

a. {0, 1, 2, 3, 4, 5}
b. {4, 5}
c. P(5) = .10
d. P(2, 3, or 4) = P(2) + P(3) + P(4) = .26 + .21 + .18 = .65
e. P(6) = 0

Problem # 3.20 For two events, A and B, P(A) = .4, P(B) = .2, and P(A ∩ B) = .1.

a. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.2} = 0.5$

b. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.1}{.4} = 0.25$

c. Events A and B are said to be independent if P(A|B)= P(A). In this case, P(A|B)=.5 and P(A)=.4 Thus A and B are not independent

Problem # 3.22

Table 3.3

	A_1	A_2
B_1	0.4	0.3
B_2	0.2	0.1

ANSWER: 6.17 P(A_1) = .4 + .2 = .6, P(A_2) = .3 + .1 = .4. P(B_1) = .4 + .3 = .7, P(B_2) = .2 + .1 = .3.

Problem # 3.23

$$P(A_1 \mid B_1) = \frac{P(A_1 \; and \; B_1)}{P(B_1)} = \frac{.4}{.7} = .57$$

$$P(A_2 \mid B_1) = \frac{P(A_2 \text{ and } B_1)}{P(B_1)} = \frac{.3}{.7} = .43$$

Did you answers to parts a and b sum to 1? Is this a coincidence? Explain
Yes. It is not a coincidence. Given B_1 the events A_1 and A_2 constitute the entire sample space.
Illustrate

Problem # 3.24 Refer to 3-26. Calculate the following probabilities.

a. $P(A_1 \mid B_2) = \frac{P(A_1 \text{ and } B_2)}{P(B_2)} = \frac{.2}{.3} = .67$

b. $P(B_2|A_1) = \frac{P(A_1 \text{ and } B_2)}{P(A_1)} = \frac{.2}{.6} = .33$

c. One of the conditional probabilities would be greater than 1, which is not possible.

Problem # 3.25

Let F = franchisees, C = company, and A = affiliates. Since A, C, and F are mutually exclusive events, P(F or A) = 21,328/31,967 + 4137/31,967 = 0.797.

Problem # 3.26

Define the following events:
A = arrested for Homicide
B = arrested for Assault
C = arrested for Burglary
D = arrested for Arson
E = arrested for Drug Offenses
F = arrested for Weapons
G = arrested for Public Disorder
H = Prosecuted
I = Convicted
J = Jailed for > 1 year

Allen was arrested for burglary, Bill was arrested for a weapons offense, and Charlie was arrested on a public-disorder charge. The decisions regarding their fates are unrelated (independent).

a. P(C and J) = 0.28

b. P[(C and I) or (F and I)]= P(C and I) + P(F and I) - P[(C and I) and (F and I)]
= 0.81 + 0.70 - (0.81)(0.70) = 0.943

c. P[(C and J') and (F and J') and (G and J')] = P(C and J')*P(F and J')*P(G and J')
= (1 - 0.28)*(1 - 0.13)*(l - 0.12) = 0.551

d. P[(C and I) and (F and I) and (G and I')]= P(C and I)*P(F and I)*P(G and I')
= 0.81*0.70*(l - 0.85) = 0.08505

e. P[(C and H') and (F and H') and (G and H')]= P(C and H')*P(F and H')*P(G and H')
= (1 - 0.88)*(l - 0.83)*(1 - 0.92) = 0.001632

f. The number of possible combinations of n = 3 with r = 1 person convicted is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \binom{3}{1} = \frac{3!}{1!2!} = 3.$$

g. The number of possible combinations of n = 3 with r = 2 people convicted is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \binom{3}{2} = \frac{3!}{2!1!} = 3.$$

h. The number of possible combinations of n = 3 with r = 3 people convicted is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \binom{3}{3} = \frac{3!}{3!0!} = 1.$$