COMM 215: BUSINESS STATISTICS

SOLUTIONS TO PRACTICE PROBLEMS 1

Descriptive Statistics

1. a.

Leaf-unit = 0.01

0	26, 45, 52, 81, 85, 88, 91, 94
1	01, 22, 42, 91, 92, 94
2	25, 33, 51, 62, 66, 93, 95, 95
3	51, 60, 72
4	91
5	19
6	71
7	96
8	81

Leaf Unit = 0.10

0	24588899
1	024999
2	23566999
3	567
4	9
5	1
6	7
7	9
8	8

b. i. $\bar{X} = 2.689$ Median = 2.29 Mode = 2.95

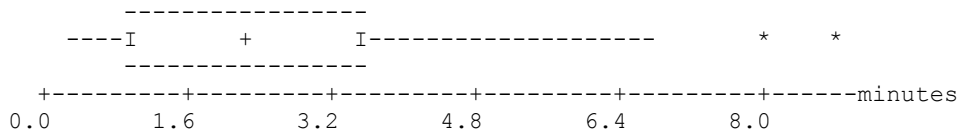
ii. Q1 = the 8^{th} ordered value = 0.94
Q3 = the 23^{rd} ordered value = 3.51

iii. Range = 8.81 - 0.26 = 8.55, IQR = 3.51 – 0.94 = 2.57

iv. $S^2 = 4.69$, S = 2.166

c. 1.5 (IQR), 1.5 (2.57) = 3.855, Q1 - 3.855 = -2.92, Q3 + 3.855 = 7.37

Two outliers whose values are 7.96 and 8.81



2. a.

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Leaf Unit = 1.0

 0 | 8
 1 | 0 5
 2 | 2 4 5
 3 | 0 5 5 8
 4 | 03
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b. $X_{min} = 8$, Q1 = 18.5, Q2 = 27.5, Q3 = 36.5, $X_{max} = 43$

c. $\bar{X} = 27.083$, S = 11.735

d. 40^{th}. Percentile = the 5^{th} ordered value = 24

3. a.

0	3		0	3 5 6 8 9
0	5 6 8 9		1	0 0 2 4 4 8
1	0 0 2 4 4	or	2	0 1 4 5
1	8		3	3
2	0 1 4			
2	5			
3	3			

b. 30^{th} percentile: 9
70^{th} percentile: 17 (.7) = 20

c. Q1 = 9, Q3 = 20. 5
IQR = 11.5
1.5(IQR)=17.25
Q3 + 1.5(IQR) = 37.75
Yes, 42 days would be considered an outlier.

4. a.

i. $\bar{X}$ = \$3430, Median = \$3500, Mode = \$3500

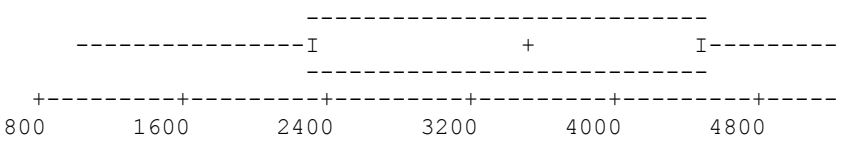
ii. Q1 = \$2,350, Q3 = \$4,450

iii. Range = 5200 - 1000 = \$4,200; IQR = 4450 - 2350 = \$2,100

iv. S^2 = 1,560,105; S = \$1,249

v.

vi.



Compute: (1.5) IQR+Q3 = 7600. No outliers

b. C.V. retainers = (1249/3430) *100 = 36.4%
C.V total compensation = (20,000/120,000) * 100 = 16.67%
No, each retainer is relatively more variable.

5. a.

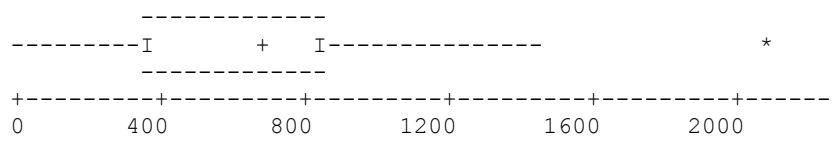
	Fi	Cumulative	Cumulative relative
20-24	3	3	0.05
25-29	7	10	0.167
30-34	11	21	0.35
35-39	13	34	0.57
40-44	9	43	0.717
45-49	7	50	0.833
50-54	5	55	0.917
55-59	3	58	0.967
60-64	1	59	0.983
65-69	1	60	1

b. Right or positively skewed.

6. a. $\bar{X}$ = \$2491.25 S= \$282.11 CV = 11.32%
b. The median = \$2405; 40^{th} percentile: = \$2390
c. CV = 320/3600 = 0.0889 * 100 = 8.89%
No, they are less variable or smaller C.V.

7. a. $\bar{X}$ = 696.809 S = 483.188

b.



IQR = 857 - 378 = 479 IQR (1.5) = 718.5, 857 + 718.5 = 1575.5
Therefore, the data value 2063 is an outlier.

c. Point where 50% of the observations are more than \$684 and 50% are less

d. The proportion below $\bar{X}$ - 1S = 213.6 plus the proportion above $\bar{X}$ + 2S = 1663.2
You would expect: 0.34 + 0.475 = 0.815 or 81.5%
Actually proportion: out of 21, there are a total of 17 in the two regions giving 17/21 = 0.809 or 80.9%.

8. a.

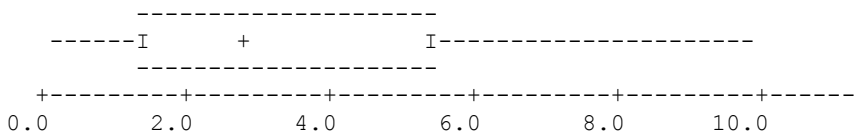
Leaf Unit = 0.10

0	2455789
1	124589
2	34457889
3	13347
4	247
5	2568
6	238
7	68
8	
9	57

b.

$X_{minimum}$ = 0.2;
Q1 = 1.45;
Q2 = 3.0;
Q3 = 5.55
$X_{maximum}$ = 9.7

c.



IQR (1.5) = 6.15; 5.55 + 6.15 = 11.7
There are no outliers.

d. Distribution is positive or right skewed.

9. a. Cov (x,y) = -15.133 indicating a negative or an inverse linear relationship between price and demand

b. r = -15.133/ ((1.94) (8.214)) = - 0.9496, indicating a strong inverse or negative linear association between price and demand.

10. Nonsampling errors occur due to incorrect acquisitions of observations, improper selections samples, and the lack of response from individual in the sample. Examples are:

1. data acquisitions: wrong measurements on employees information (*answers may vary*)
2. exclusion of part-time employees from the sample (*answers may vary*)
3. missing information on employees characteristics; responses coming only from people who have strong feeling about the issues (*answers may vary*)

Probability

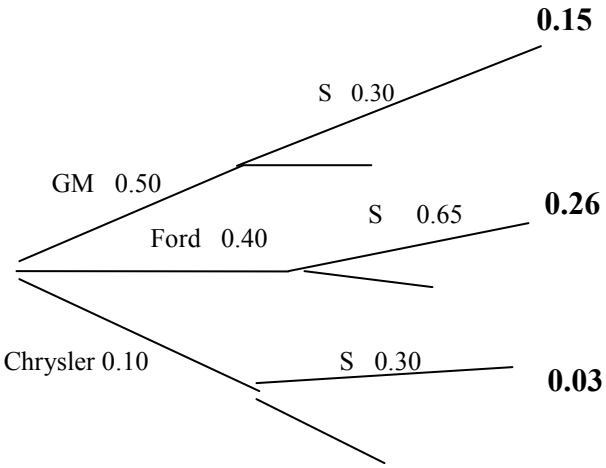
1.

	E	**U**	
M	.52	.08	**.60**
W	.28	.12	**.40**
	.80	**.20**	

.6 U = WU
.6 (.2) = .12
.3W = WU,
.3W = .12,
W = .4

a. P(W) = 0.40
b. P(U/M) = 0.08 / 0.60 = .13

2.

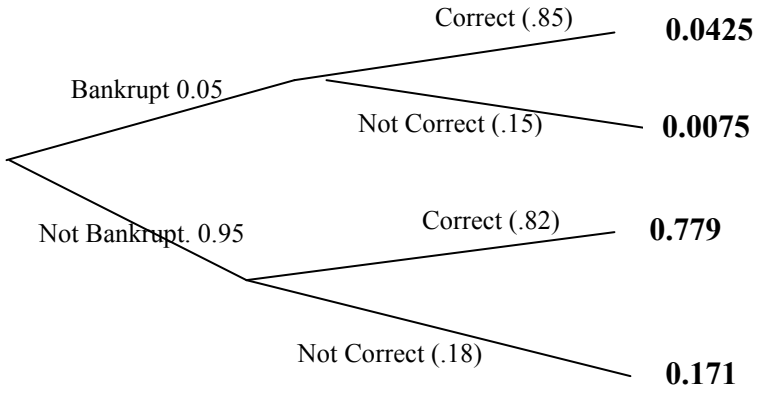


P (Strike) = .15 + .26 + .03 = .44

3.

a. (.9) (.9) = .81
b. (.1) (.1) = .01
c. (.9) (.1) + (.1) (.9) + (.9) (.9) = .99 or 1 – 0.01 = 0.99

4.



P (correct) = 0.0425 + 0.779
= 0.8215
0.8215 * 100 firms = 82 firms

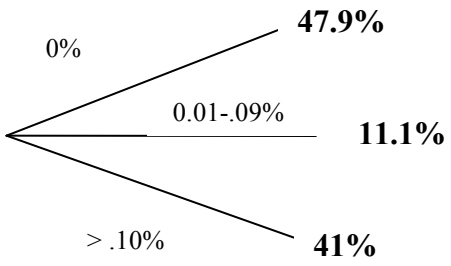
5.

a. P(<30 or 1st) = 1 – (.07 + .09 + .05) = .79
b. P(repeat | < 30) = (.01+.02+.04)/.43 = .16
c. P (≥ 30 | 1st) = .36/.72 = .50
   P (≥ 30 | repeat) = .21/.28 = .75; repeat offender is more likely to be ≥ 30

d. 50th p of first offenders = .36 = 30
   a. 50th p of repeat offenders = .14 = 35, repeat offenders are older

e. (.28) (.28) = .0784

6.



a. P (X≥ .01) = 11.1 + 41 = 52.1%
b. $P(X>0.1 \mid X \geq 0.01) = .41 / .521 = .7869$

7. a. i. 365/500 = .73 iii. 30/500 = .06

ii. 110/340 = .3235 iv. $\frac{230+140-30}{500} = .68$

b. Not mutually exclusive since P (work pt-time and < 6) = 60/500 = .12.
It will be zero if mutually exclusive.

Independent if P(work pt. time) * P(<6) = P(work pt. time and < 6)

Since 135/500 * 140/500 ≠ .12, the two are not independent.

8.

	P	S	A	
U	0.125	0	0.06	0.185
U^C	0.125	0.15	0.54	0.815
	0.25	0.15	0.60	1

a. P(U) = 0.185
b. $P(A|U^c) = .54/.815 = .6625$

9.

a.

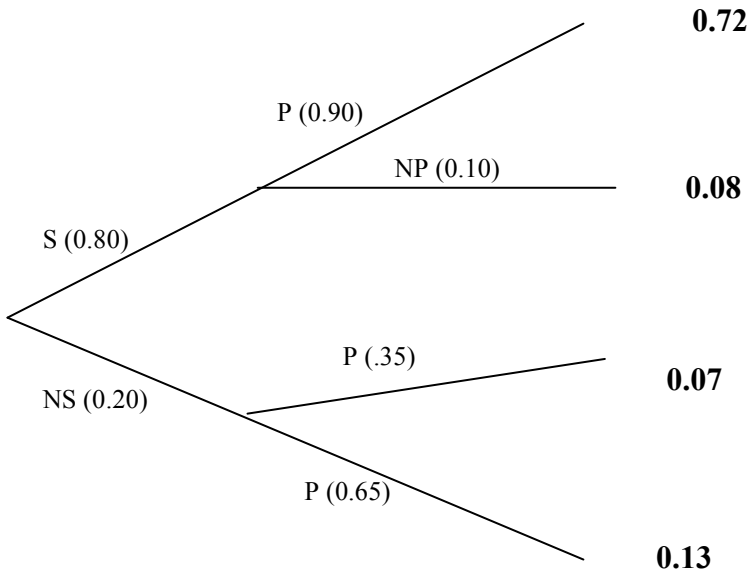
Own Stock	City			
	A	B	C	Total
YES	85 (.17)	55 (.11)	50 (.10)	190 (.38)
NO	115 (.23)	95 (.19)	100 (.20)	310 (.62)
Total	200 (.40)	150 (.30)	150 (.30)	500 (1.0)

b. i. P(YES) = 0.38; ii. P(YES and A) = 85/500 = 0.17;

iii. P(B and NO) = 95/500 = 0.19; iv. P(B and YES) = 215/500 = 0.43

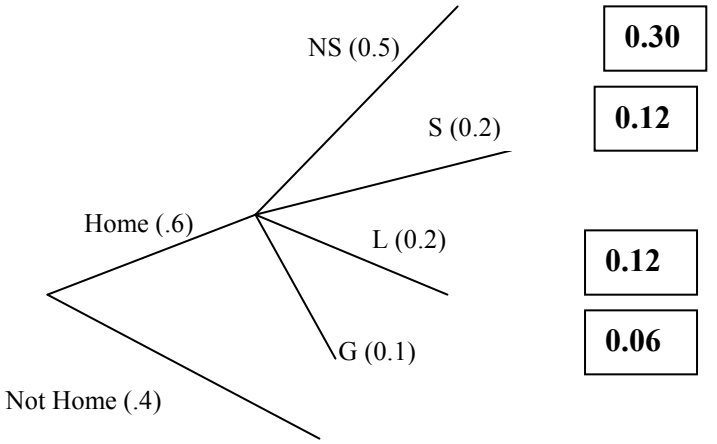
c. P (NO / NOT B) = 215/350 = 0.614

10.



a. P(S/P) = .72/.79 = 0.91 **b. P(NS/NP) = 0.61**

11.



a. 0.4 + (.6) (.5) = .70
b. 0.12 + .06 = .18

12.

a.

	Defective (D)	**Not Defective (ND)**	
1	.008	.392	.40
2	.0075	.2925	.30
3	.0045	.2955	.30
	.02	.98	

b. P (1/D) = .008/.02 = .40
c. P (D or 2) = .02 + .03 - .0075 = .3125
d. P (D) = .02
e. P (2/D) = P (2) = .0075/.02 = .375 = .30; not independent

13. a.

	E	U	
M	.5	.04	.54
F	.43	.03	.46
	.93	.07	1

b. P(F/U) = .03/.07 = .428
c. i. P(um*um) = (.04) (.04) = .0016 ii. P(1 or 2)

Easiest way would be with Binomial (but not the only way)

(Combination of 2 and 1) (.04) (.96) + (Combination of 2 and 2) $(.04)^2$ $(.96)^0$ = .0784

d. P(U) = .07
e. P(Males) = .54 P(Male/U) = 0.04/0.07 .54 ≠ .571

Therefore they are not independent

(Can use any of the formulas as long as it is correct)

$P(A \cap B) = P(A) * P(B)$ P(A/B) = $\frac{P(A \cap B)}{P(B)}$ P(B/A) = $\frac{P(A \cap B)}{P(B)}$

Discrete Probability Distribution / Binomial Distribution

1. a.

	1st	2nd	3rd	Total
Regular	.9 (.2)=0.18 $4.00	.7 (.3)=0.21 $2.00	.8(.5)=0.4 $0.50	0.79
Reduced	.1 (.2)=0.02 $2.00	.3 (.3)=0.09 $1.00	.2 (.5)=0.10 $0.25	0.21
Total	0.20	0.30	0.50	1

P (regular) = .79

b. $P(1^{st}/\text{regular}) = .18/.79 = .227$

c. $\mu = .18\,(4) + .21\,(2) + .4\,(.50) + .02\,(2) + .09\,(1) + .10\,(.25)$
$= \$1.495$

$$\mu_{regular} = \frac{.18(4)+.21(2)+.4(.5)}{.79} = \$1.696$$

d. $P(X \geq 2 / n = 8, p = .21) = 1 - \{f(0) + f(1)\}$

$P(0) =$ (Combination of 8 and 0) $(.21)^0\,(.79)^8 = .1517$
$P(1) =$ (Combination of 8 and 1) $(.21)^1\,(.79)^7 = .3226$

$1 - (.1517 + .3226) = .5257$
$E(X) = 8\,(.21) = 1.68;$ $\sigma = \sqrt{1.68(.79)} = 1.15$

e. $\$1.495\,(75000) = \$112{,}125$ or $\$1.50\,(75000) = \$112{,}500$

2. a. X = arrive on time p = .75
$P(X = 4 / n = 4, p = .75) = .3164$

b. $P(X = 3) + P(X = 4) = .4219 + .3164 = 0.7383$

3. $.05\,(2) + .15\,(5) + .25\,(10) + .35\,(20) + .10\,(50) + .10\,(100)$

$$\mu = \sum Xi * P(X) = \$25.35, \qquad \sigma^2 = \Sigma(X - \mu)^2 P(X) = 776.33$$

4. a. $\mu = np = 12(.30) = 3.6\,people$ $\mu = np = 12(.30) = 3.6\,people$

b. $P(X \geq 3 / n = 12, p = .30) = 1 - (f(0) + f(1) + f(2))$
$1 - (.0138 + .0712 + .1678) = .7472$

c. $P(X \geq 4 / n = 12, p = .30) = 1 - (f(0) + f(1) + f(2) + f(3))$
$1 - .4925 = 0.5075$

5. a. $E(X) = .3\ (100) + .2\ (100) + .7\ (100) = \$120$

b. Var (X) = $\sum (X - \mu)^2 P(x) = \$480.00$

6.

a. $\sum (X - \mu)^2 P(x) = \$480.00$

$P(X=3 / n = 15, p = .05) =$ (Combination of 15 and 3) $(.05)^3\ (.95)^{12} = .0307$

b. $P(X \geq 3 / n = 15, p = .05) = 1 - (f(0) + f(1) + f(2)) = 0.0361$

c. If P is actually .05, than $P(X \geq 3)$ is very small therefore it is unlikely.

7.

a. $E(X) = \mu = 10\ (.2) = 2$; standard deviation = $\sqrt{10(.2)(.8)} = 1.6$

b. $P\ (X \leq 2 / n = 10, p = .2) = .1074 + .2684 + .3020 = .6778$

c. $P(X=3 / n = 10, p = .8) =$ (Combination of 10 and 3) $(.8)^3\ (.20)^7 = .000786$

d. $p = .2/(1-.2) = 0.25$ $P\ (X \leq 2 / n = 9, p = .25) = 0\ .60$

8.

a. Mean = $\sum Xp(x) = 1.98$

b. Var(x) = $\sum (x - \mu)^2\ p(x) = 1.919$ Standard deviation = 1.385

c. Expected Profit = 1.98 (2000) = \$3,960

d. $P(X_1 + X_2 < 3) = P\{(0,0), (0,1), (0,2), (1,0), (1,1), (2,0)\}$
$= (.15)(.15) + (.15)\ (.24) + (.15)(\ .31) + (.24)\ (.15) + (.24)\ (.24) + (.31)\ (.15) = .2451$

9.

a. $P(X = 6 / n = 12, p = .30) = .0792$

b. $P(X \geq 6) = 1 - P(X \leq 5) = .1179$

c. $P(X \leq 2) = f(0) + f(1) + f(2) = .2528$

d. $E(X) = np = 12\ (.3) = 3.6,$ or $3.6\ (100) = \$360$

10.

a. $E(X) = 0\ (.48) + 1\ (.2) + 2\ (.15) + 3\ (.08) + 4\ (.05) + 5\ (.03) + 6\ (.01) = 1.15,$ $Var(X) = 2.1075$

b. $f(2) + f(3) = .15 + .08 = .23$

c. $P(X \geq 2) = 1 - (.48 + .20) = .32$

d. $P(X<3) = P(0,0) + P(0,1) + P(0,2) + P(1,0) + P(1,1) + P(2,0) = .6064$

11.

a. $P(X \geq 3) = 1 - \{f(0) + f(1) + f(2)\} = 0\ .74$

b. $P(2 \leq X \leq 5) = f(2) + f(3) + f(4) + f(5) = 0\ .67$

c. $E(X) = 0\ (.05) + 1\ (.08) + 2\ (.13) + 3\ (.23) + 4\ (.18) + 5\ (.13) + 6(.08) + 7\ (.06) + 8\ (.06) = 3.78$

d. $\sigma = \sqrt{4.2} = 2.04$

12.

a. E(X) = 0 (.1) + 1000 (.3) + 2000 (.4) + 3000 (.2) = $1700

b.

Sales (x)	Net Profit	P(X)	
0	-28500	.1	-2850
1000	-12500	.3	-3750
2000	3500	.4	1400
3000	19500	.2	3900
Expected Net Profit			**$-1300**

13.

a. P(X≥9 / n = 15, p = .7) = 1 – P(X≤8) = 1 - .131 = 0.869
b. P(X≥12 / n = 15, p = .7) = 1 – P(X≤11) = 1 - .703 = 0.297
c. P(X≤2 / n = 15, p = .25) = p(0) + p(1) + p(2)
= .0134 + .0668 + .1559 = 0.236
d. 0.297 (250) = 74.25

14.

a. p(1) + p(4) + p(5) = .25 + .15 + .10 =0 .5
b. 1 (.25) + 2(.33) + 3(.17) + 4(.15) + 5(.10) = 2.52
c. $\sigma = \sqrt{1.649} = 1.28$

15.

a. P(X=10 / n = 10, p = .6) = (Combination of 10 and 10) $(.6)^{10}$ $(.4)^{0}$ = .006
b. P(X≤2) = p(0) + p(1) + p(2) = .0001 + .0016 + .0106 = .0123

1 a. $p(x \le 60)=p\left(z \le \frac{60-80}{10}\right) = p(z \le -2) = .0228$

b. $p(60 \le x \le 75)=p\left(\frac{60-80}{10} \le z \le \frac{75-80}{10}\right) = p(-2 \le z \le -0.5)$

c. $p(x \ge 90)=p\left(z \ge \frac{90-80}{10}\right) = p(z \ge 1) = 0.1587 \times 60 = 9.5 \approx 10 student$

2 a. $p(x \ge 4)=p\left(z \ge \frac{4-4.5}{0.6}\right) = p(z \ge -0.833) = 0.5 + 0.2967$

$0.7967 \times 80 = 63.7 \approx 64$

b. $10/80 = 0.125, \quad p(z \ge z_0) = .125 \rightarrow z_0 = 1.15$

$4.5 + 1.15(0.6) = 5.19$

c. $p\left(z \ge \frac{5.1-4.5}{0.6}\right) = p(z \ge 1)$ and $p\left(z \ge \frac{4.5-4.5}{0.6}\right)$

$p\left(\frac{z \ge 1}{z \ge 0}\right) = \frac{.1587}{.5} = 0.317$

3 a. $p(x>36.000)$ or $p(x<33.500)=p\left(z>\frac{36.000-34.265}{2000}\right)+$

$p\left(z < \frac{33.500-34.265}{2000}\right) = p(z > 0.861) + p(z < -0.383)$

$0.192 + 0.352 = 0.544$

b. $p\left(z>\frac{36.000-34.265}{2000}\right) \times p\left(z > \frac{36.000-35.185}{2400}\right)$

$p(z \ge 0.868) \times p(z > 0.34) = (.192)(.3669) = 0.07$

c. $p(x \le x_0) = .90 \quad 1.28=\frac{x-35.185}{2400} \quad x=\$38.257$

4 a. $1-p(18 \le x \le 26)=1-\left(\frac{18-20}{5} \le z \le \frac{26-20}{5}\right)$

$1 - p(-0.4 \le z \le 1.2) = 1 - (0.3849 + 0.1554) = 0.459$

b. $p(x \le 7)=p\left(z \le \frac{7-20}{5}\right) = -2.6 \quad p(z \le -2.6)=0.0047$

$\frac{p(x \le 7)}{p(x \le 20)} = \frac{0.0047}{0.5} = 0.0094$

4. c. $p(x \geq 1) = 1 - p(x - 0)$

$$p(x = 0) = \frac{10!}{0!10!}(.54)^{0}(.46)^{10} = 0.000424$$

$1 - 0.000424 = 0.999576$

d. $p(x \leq 15) = p\left(z \leq \frac{15 - 20}{5}\right) = p(z \leq 1) = 0.5 - 0.3413$

$= 0.1587 \times 200 = 31.74 \approx 32$

5. a. $p(x > 140.5) = p\left(z > \frac{140.5 - 140}{0.2}\right) = p(z > 2.5) =$

$.05 - 0.4938 = 0.0062$

b. $p(140.2 \leq x \leq 140.5) \quad z\left(\frac{140.2 - 140}{0.2}\right) = 1.0$

$p(1.0 \leq z \leq 2.5) = 0.4938 - 0.3413 = 0.1525$

c. $p(x < 139.8) \ or \ p(x > 140.2)$

$p(z < -1) \ or$ p(z>+1)=0.5-0.3413=0.1587 × 2=0.3174

d. $p(x = 0 / n = 15, p = 0.3174) = \frac{15!}{0!15!}(0.3174)^{0}(0.6826)^{15} = 0.0033$

e. $p(x = 5 / n = 15, p = 0.3174) = \frac{15!}{5!10!}(0.3174)^{5}(0.6826)^{10} = 0.2110$

6. a. $p(x \leq 6) = z\left(\frac{6 - 12}{3}\right) = -2.0 \quad p(z \leq -2) = 0.5 - 0.4772 = 0.0228$

b. $p(x > 15) = p\left(z > \frac{15 - 12}{3}\right) = p(z > 1.0) = 0.5 - 0.3413 = 0.158$

c. $z = \frac{x - \mu}{\sigma} \quad 1.28 = \frac{x - 12}{3}$ x=15.84 ≈ 16weeks

7. a. $p(x>214)=p\left(z> \frac{214-250}{45}\right) = p(z > -0.8) = 0.2881 + 0.5 = 0.7881$

b. $p(x \leq 200) = p\left(z \leq \frac{200 - 250}{45}\right) = p(z \leq -1.11) = 0.5 - 0.3665 = 0.1135$

Thus, 11.35% of 52 weeks. That is, $0.1135 \times 52 weeks = 6.9 \approx 7 weeks$

c. Find the 60th percentile, such that $P(Z \leq z^*) = 0.6$,

where $z^* = \frac{x^*-250}{45}$. From table, $z^*=0.2533$. Then, $0.2533 = \frac{x^*-250}{45}$,

and solving for $x^*=0.2533*45+250=261.4006$