Concordia University
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COMM 215 Business Statistics
Section BX

Final Examination
August 16, 2013

Last Name: ______________________________ **First Name**______________________
(Please Print)

Student No.:______________________

INSTRUCTIONS

1. **Attempt all questions**. Show your work for FULL credit.
2. This is a **closed book, closed note** examination. You are allowed to use standard, basic calculators during the examination. Sharing of calculators is not allowed. **DO NOT DETACH pages**. Return the exam booklet intact.
3. For **PART I** - **Multiple Choice Questions:**
   a. Use **PENCIL** to **fill** the appropriate circles on the blue sheet corresponding to the correct **answer choices.**
   b. Use **PENCIL** to **write** your full name and Student ID, and to **fill** the matching circles below your name and ID on the blue sheet.
4. For **PART II** – **Problem Solving:**
   a. Use **PEN to write your answers in the space provided** below each question.
   b. You may use both sides of the paper if necessary. **Do not include extra pages.**
5. Tables and formulas are appended. **Do not DETACH from the booklet.**
6. **No questions** about the examination are allowed.

		Marks obtained
Part I	Sub-Total	/48
Part II		
	Question 1	/5
	Question 2	/10
	Question 3	/10
	Question 4	/15
	Question 5	/12
	Sub-Total	/52
Total		100

PART I: Multiple choice questions **(One mark for each of questions 1 to 16 and two marks for each of questions 11 to 32). Some of the numbers in the provided choices have been rounded.**

Circle the correct answer completely.

1. The value of the standard error of the mean
   a. is always larger than the population standard deviation
   b. is always equal to the population standard deviation
   c. is always smaller than the population standard deviation
   d. could be larger, equal to, or smaller than the population standard deviation
   e. None of the suggest answers are correct

2. A sample of 200 students from a School of Business resulted in 35% of the students to be in the age group 17 to 20, 30% in age group 21 to 24, 20% in age group 25 to 28, and 15% in age group 29 to 31. Which of the following is true?

   a. The data are on ordinal scale since the age groups can be ordered according to their relative frequencies
   b. A scatter diagram is an appropriate graphical representation of the data since it displays the relationship between relative frequency and age group
   c. A pie chart can be used to represent the data since the data collected are categorical data
   d. None of the suggest answers are correct

3.The median is used as a measure of central tendency since
   a. it can be computed if the data have a mix of positive and negative values
   b. it cannot be influenced too much by extreme values
   c. it will have a reasonably small value if the smallest observation is small
   d. it will have a reasonably large value if the largest observation is large
   e. None of the suggest answers are correct

4. A histogram is
   a. the same as a pie chart
   b. used to represent categorical data as it displays graphically the frequencies or relative frequencies of all the categories
   c. a graphical method of presenting a cumulative percentages
   d. in some ways similar to the stem plot
   e. None of the suggest answers are correct

5. Which of the following is incorrect?
   a. The range is a measure of variation that is less affected by extreme values than the interquartile range
   b. The median is a measure of central tendency that is less affected by extreme values than the mean
   c. The mode is a measure of central tendency that is less affected by extreme values than the mean
   d. The interquartile range a measure of variation that is less affected by extreme values than the standard deviation
   e. None of the suggest answers are correct

6. Which of the following best describes the form of the sampling distribution of the sample proportion?
   a. When standardized, it approximates the binomial distribution.
   b. When standardized, it is the t distribution.
   c. When standardized, it is the normal distribution.
   d. When standardized, it approximates the normal distribution regardless of the sample size.
   e. None of the suggest answers are correct

7. Fifty-five percent of the graduates of a medical schools last year intended to specialize in family practice. If a sample of more than 36 graduates is taken, then the sample proportion of graduates who intended not to specialize in family practice
   a. must be equal to 0.55
   b. has a mean that is equal to 0.55
   c. has a standard deviation that is equal to 0.0829
   d. has a standard deviation that is equal to 0.0413
   e. None of the suggest answers are correct

8. Which of the following statements is false?
   a. If the 95% confidence interval for the number of moviegoers who purchase from the concession stand is 30% to 45%, then fewer than half of all moviegoers do not purchase from the concession stand.
   b. Errors in the simple linear regression model represent the net effects on the response of other variables that are not accounted for by the model.
   c. Prediction intervals get wider as you extrapolate outside the range of the data.
   d. The use of a linear equation to describe the association between price and sales implies that we expect equal differences in sales when comparing periods with price $10 and $11 and periods with price $20 and $31.
   e. None of the suggest answers are correct

9. Which of the following statements is always true regarding a simple linear regression model?
   a. The proportion of variation explained is the correlation coefficient
   b. Explained variation must be greater than SSE if the correlation coefficient is larger than 0.5
   c. The slope coefficient must have the same sign as the coefficient of determination
   d. The slope coefficient must have the opposite sign as the coefficient of determination
   e. None of the suggest answers are correct

10. Which of the following statements is true?
    a. If the correlation between the growth of a stock and the growth of the economy as whole is close to 1, then this would be a good stock to hold during a recession when the economy shrinks.
    b. The correlation between x and y is 1, the points in the scatterplot of y on x all lie on a straight line with slope s_x / s_y.
    c. A gasoline station kept data on its monthly sales over the past 3 years along with the average selling price of gasoline. A scatterplot of sales on price of gasoline showed positive association. This association means that higher prices lure more customers.
    d. By increasing the sample size from n=100 to n=400, we can reduce the margin of error by 50%.
    e. None of the suggest answers are correct

11. The width of a confidence interval estimate of the population mean will increase as

a. The sample size increases
b. The confidence coefficient decreases
c. The standard error increases
d. The confidence level increases
e. None of the suggest answers are correct

12. If σ^2 doubles, the width of the confidence interval for the population mean, assuming normality will

a. double
b. quadruple
c. be multiple of $\sqrt{2}$
d. decrease
e. None of the suggest answers are correct

13. From a random sample of 200 residents of a large city, a 95% confidence interval estimate for the proportion of residents who favored an increase of funding to improve the public transit is found to be (.795 .895). Which of the following is the correct interpretation of the interval?
    a. The proportion of residents in the city who favored an increase in funding must be between 0.795 and 0.895
    b. We are 95% confident that this interval contains the true proportion of residents who favor funding for improvements of the public transit
    c. Only 5% of the residents would not favor an increase in funding
    d. The interval cannot be interpreted since the margin of error cannot be found with the given information
    e. None of the suggest answers are correct

14. A random sample of size 49 is taken from an infinite population that has a mean of 80, and a variance of 196. The distribution of the population is unknown. The standard deviation of the sample mean
    a. can only be estimated if all possible samples of size 49 are given
    b. can only be estimated if the sample mean is close to the population mean
    c. is equal to 28
    d. is equal to 2
    e. None of the suggest answers are correct

15. An agricultural company has determined that the weight of hay bales is normally distributed with a mean equal to 100 pounds and a standard deviation equal to 10 pounds. Based on this, what is the probability that the mean weight of the bales in a sample of 25 bales will be between 98 and 102 pounds?

a. 0.1586
b. 0.5793
c. 0.4207
d. 0.6826
e. None of the suggest answers are correct

16. Which of the following statements is true?
    a. If we select several random samples of the same size from a normally distributed population and we compute the sample means, they must all have the same values
    b. If we select several random samples from a normally distributed population, the expected values of these sample means will be the same regardless of the sample size
    c. If we select several random samples of the same size from a normally distributed population, the sample means will have different standard deviations.
    d. None of the suggest answers are correct

17. A major shipping company claimed that 96 percent of all parcels are delivered on time. To check this, a random sample of 400 parcels was taken. Of these, 368 arrived on time. If the company's claim is correct, what is the probability of 368 or more parcels arriving on time?
    a. 0.0745
    b. 0.0139
    c. 0.9255
    d. None of the suggest answers are correct

18. The Bureau of Labor Statistics reported that the average yearly income of dentists in the year 2009 was \$110,000. A sample of 81 dentists taken in 2010 showed an average yearly income of \$120,000. Assume the standard deviation of the population of dentists in 2010 is \$36,000 and the distribution of income is approximately normally distributed. The p-value for testing if the average income in 2010 has increased from 2009 is found to be to 0.0062. Which of the following is true?
    a. The p-value cannot be interpreted since the significance level is not given
    b. The probability that the sample mean is greater than \$110,000 is only 0.0062
    c. The p-value can be interpreted as the probability of obtaining a sample with a z value larger than 2.5 if the average income has changed
    d. None of the suggest answers are correct

19. An official of a large national union claims that the fraction of men in the union is more than one-half. A sample of 100 employees is selected and 58 of them are men. A 1% level of significance is used to test the union's claim. Which of the following statements is true?
    a. The null hypotheses should be $H_0: p \geq 0.58$ since the sample proportion is less than 0.58
    b. The correct hypotheses are: $H_0: p \leq 0.58$ and $H_a: p > 0.58$
    c. The correct hypotheses are: $H_0: p = 0.5$ and $H_a: p \neq 0.5$
    d. The correct hypotheses are: $H_0: p \leq 0.5$ and $H_a: p > 0.5$
    e. None of the suggest answers are correct

20. Last year, 21% of people prefer the soft drink of a manufacturer to those of its competitor. The manufacturer is interested in investigating if this figure has increased. A sample of 400 individuals participated in the taste test and 100 said they prefer the manufacturer's soft drink to those of its competitors. Hypotheses test is to be conducted at 5% level of significance. Which of the followings is true?
    a. The p-value is P(Z > 1.96) = 0.025
    b. The p-value is P(Z > 1.85) = 0.0322
    c. The p-value is 2P(Z > 1.96) = 0.05
    d. None of the suggest answers are correct

21. Suppose that the null hypothesis is rejected at 5% level in a hypothesis test with a one-sided alternative hypothesis H_a: $\mu < 32$ .Which of the following statements is incorrect?
   a. The p-value must be smaller than 0.025 since it is a one-sided test
   b. There is sufficient evidence at 5% level that the mean is less than 32
   c. The test statistic value is less than the rejection point of the test
   d. None of the suggest answers are correct

Refer to the following in answering questions 22 to 25

An occupant traffic study was carried out to aid in the remodeling of a large building on a university campus. The building has 5 entrances (A, B, C, D and E), and the choice of entrances was recorded for a random sample of 300 people entering the building, yielding the frequency distribution below. The manager would like to test at 5% level if the 5 entrances are equally used.

Entrance	Number of people using
A	79
B	51
C	52
D	69
E	49

22. After analyzing the data with a Chi-Square test, a statistician reported that there is no evidence to reject the null hypothesis at 5% level. Which of the following is correct?
   a. A conclusion should not be drawn from the Chi-Square test since some expected frequencies are less than 5.
   b. There is evidence at 5% level that the entrances are not equally used
   c. There is evidence at 5% level that the entrances are equally used
   d. None of the suggest answers are correct

23. Which of the following statements is true?
   a. All expected frequencies are smaller than observed frequencies since the sample size is large.
   b. A chi-square test is appropriate since the observed frequencies are all bigger than 5.
   c. A test based on the standard normal distribution is more appropriate since the sample size is larger than 30.
   d. The sum of the expected frequencies are equal to the sum of the observed frequencies assuming that the entrances are equally used
   e. None of the suggest answers are correct

24. A statistician uses the Chi-Square statistic to test hypotheses. The sum of the expected frequencies for entrances A and E in the calculation of the Chi-Square statistic is
   a. equal to 128, assuming that the entrances are equally used
   b. equal to 128, assuming that the entrances are not equally used
   c. equal to 120
   d. None of the suggest answers are correct

25. A statistician uses the Chi-Square statistic to test hypotheses and the chi-square statistic is found to be 11.8. Which of the following is a correct statement?
   a. The p-value is between 0.02 and 0.05 since two tail rejection points should be used.
   b. The p-value is between 0.025 and 0.05 for one-tail and two-tail rejection points
   c. The p-value is above 0.01 but below 0.025
   d. None of the suggested answers are correct

26. Which of the following is not indicative of a negative relationship between X and Y?
   a. A negative correlation coefficient
   b. The slope is negative
   c. A very high value of the coefficient of determination
   d. None of the suggested answers are correct

27. When comparing the 90 percent prediction interval and 99% confidence interval for a mean (with the same given x) for simple linear regression analysis, which of the following is true?
   a. The prediction interval is always narrower than the confidence interval.
   b. The prediction interval is always wider than the confidence interval.
   c. The prediction interval may be wider or narrower than the confidence interval, depending on the value of x
   d. The two intervals are often identical since they are based on the same predicted value
   e. None of the suggest answers are correct

Refer to the following in answering questions 28 to 30

A financial consultant was interested in using simple linear regression to study the relationship between the annual percentage change in stock price y and the annual percent change in profits x. The following data was determined for 7 randomly selected companies:

% change in stock price	% change in profit
8.4	4.2
9.5	5.6
13.6	11.2
-3.2	4.5
7	12.2
18.4	12
-2.1	-13.4

You are given that SSE = 188.1 and explained variation = 179.5. The sample regression line is found to be $\hat{y} = 4.197 + 0.6122x$.

28. The proportion of variation in stock price percentage change explained by the percent change in profits
   a. cannot be determined since the sample shows that the change in price can be negative for positive change in profit
   b. is greater than 0.5 since SSE is greater than explained variation
   c. is 0.4883
   d. None of the suggest answers are correct

29. A company not in the sample has a 6.5 percentage increase in profit, the predicted percentage change in stock price

a. cannot be determined since the sample shows that the change in price can be negative for positive change in profit
b. is 6.95
c. is 8.18
d. is approximately the same as when sample has a 4.5 percentage increase in profit
e. None of the suggest answers are correct

30. Which of the following statements is true regarding the value of the test statistic for testing $H_0: \beta_1 \leq 1$ against $H_a: \beta_1 > 1$?

a. The standard error of the estimate is $s = 6.134$ and the value of the t test statistic is therefore $\frac{0.6122}{6.134/\sqrt{7}} = 0.264$
b. the t test statistic is only good for a two-tailed test since change in stock price can be both positive and negative
c. The t test statistic cannot be determined without knowing the significance level
d. None of the suggest answers are correct

31. A Dépanneur maintains a Web site that it uses to attract shoppers. The average purchase amount is $80. The store is evaluating a new Web site that it hopes would encourage shoppers to spend more. Let μ represent the average amount spent per customer at its redesigned Web site. Which of the following statements is true?

a. The appropriate null hypothesis for testing the profitability of the new design sets $H_0: \mu = 80$.
b. If the level of significance is 5%, then there is at most a 5% chance of incorrectly rejecting H_0.
c. If the test used rejects H_0 at 5% level of significance, then it would also reject H_0 at 1% level of significance.
d. If the standard deviation is estimated from the data, then z-statistic determines the p-value.
e. None of the suggest answers are correct

32. A regression analysis resulted in the following equation:

$$\hat{y} = 145.3 + 25X_1 - 5.7X_2 + 0.82X_3$$

If each of X_1 and X_2 decreases by 1 and X_3 increases by 2, then the expected value of y changes by the estimated amount of

a. zero
b. -37.22
c. -17.66
d. Can't be determined since the independent variables change simultaneously
e. None of the suggest answers are correct

Part II

Question 1 (5 marks)

A food store has determined that daily demand for milk cartons has a normal distribution, with a mean of 55 cartons and a standard deviation of 6 cartons.

a. On Saturdays, the demand for milk is known to exceed 60 cartons. On the coming Saturday, what is the probability that it will be at least 70 cartons?

b. The store receives 50 cartons of milk each morning, from which one is put aside for the manager's personal use. What is the probability of having an insufficient number of milk cartons to meet demand?

Question 2 (10 marks)

The Federal Environmental Agency has warned the city of about recurring poor air quality indicators due to the presence of an air pollutant called Carbon Monoxide (CO). CO is a colorless poisonous gas that is emitted directly from automobile tailpipes. Some years ago, the Agency imposed limits of 2.1 g/km on exhaust gas emissions at the tailpipe. The city is planning to launch a massive campaign of gas emission controls. A prior pilot set of controls was achieved on a random set of 30 cars.

2	2.03	1.55	1.38	2.58	1.62	0.63	1.14	1.69	0.88
1.9	2.98	2.39	1.07	2.89	0.51	2.69	2.04	2.32	1.99
1.24	0.08	1.49	1.91	2.92	1.7	2.03	1.36	2.49	1.62

$\sum x_i = 53.12, \sum x_i^2 = 109.359$

a. Estimate with a 95% confidence interval, the true mean CO emission per car. Interpret.

b. The city of wants to know with a confidence level of 90% the true mean CO emission per car with a margin of error of maximum 0.025. Assuming that the population standard deviation is unknown, how many additional cars should be tested to provide such an estimate?

Question 3 (10 marks)

A travel company wishes to determine if the type of vacation purchased in its market area is independent of income level of purchasers. A random survey of purchasers gave the following results:

Vacation Type	Income Level		
	High	Medium	Low
Domestic	50	120	65
Foreign	25	30	10

a. At the 0.05 level of significance, can it be concluded that vacation preference and income level are statistically independent? Interpret the result in the context of the problem.

b. At 5% level of significance, is there sufficient evidence to conclude that more than a third of the vacationers are high-income purchasers. Interpret the result in the context of the problem.

Question 4 (15 marks)

ZUBRAK Inc. is a computer firm specializing in Web designs. Since ZUBRAK uses its own special web page design software, all newly hired employees have to go through a training course regardless of their work experience in computing. The vice-president of personnel is skeptical about paying a higher salary to hire more experienced people since they have to be retrained. For each of 15 randomly selected new employees, she obtained data on the employee's score in web page design skill Y (after the training course) and also the number of months of computer related work experience X at the time of hiring. Part of the data along with sums, sums of squares and some relevant statistic are given below.

Employee	1	2	3	.	.	.	13	,14	15
X_i	19	10	12	.	.	.	18	15	14
Y_i	81	66	70	.	.	.	88	78	73

$$\bar{X} = 15.733,\ \bar{Y} = 76.4667,\ \sum(X_i - \bar{X})^2 = 164.934, \sum(Y - \bar{Y})^2 = 9376.22,\ SSE = 188.03,\ S_{b_1} = 0.2961.$$

Suppose that a simple linear regression model is appropriate for analyzing the above data and the least squares fit is obtained as $\hat{Y}_i = 49.579 + 1.709X_i$,

a. At 5% significance level, is there sufficient evidence that design skill is positively related to computer related work experience? Interpret the result in the context of the problem.

b. What proportion of the variation of design skill scores is accounted for by computer related work experience? Interpret the result in the context of the problem.

c. Find a 95% prediction interval for design skill score of an employee with 15 months of computer related work experience. Interpret the result in the context of the problem.

Question 5 (12 marks)

Based on a survey of 20 firms, a researcher has developed a multiple regression model relating sales (**SALES** in $1,000) to inventory investment (**INVEST** in $1,000), advertising expenditures (**AD** in $1,000) and the average bonus paid to employees (BONUS in $1,000). The table below shows partial results obtained from fitting a multiple regression model using Excel.

Regression output				
	Coefficients	*Std. error*	*t*	*p-value*
Intercept	25.50	18.802		
INVEST	10.05	4.251		
AD	8.05	3.502		
BONUS	0.125	0.041		

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Regression					
Residual	3360				
Total	16800				

a. Is there sufficient evidence at the 5% level of significance to conclude that the model is useful in predicting sales? Interpret the result in the context of the problem.

b. Is there sufficient evidence at the 5% level of significance to conclude that sales are related to expenditure on advertisement, given that inventory investment and bonus paid to employees remain unchanged? What is the p-value of the test?

c. Estimate the coefficient of determination and explain its meaning in the context of the problem. Interpret the result in context of the problem.

d. Estimate sales based on a planned investment in inventory of $15,000, an advertising budget of $10,000 and an average bonus to employees of $2,000 for the coming year.

COMM 215 Business Statistics
List of formulae provided in the Final Examination

Chapter 2 Descriptive Statistics

Sample mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Sample variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]$$

Sample standard deviation: $s = \sqrt{s^2}$

Z score: $z = \dfrac{x - \text{mean}}{\text{standard deviation}}$

Coefficient of variation: $\dfrac{\text{standard deviation}}{\text{mean}} \times 100$

Chapter 3 Probability

The rule of complement: $P(\bar{A}) = 1 - P(A)$

The addition rule for two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

The general multiplication rule:

$$P(A \cap B) = P(A)P(B \mid A)$$

Chapter 4 Discrete random variables

Mean (expected value) of a discrete random variable

$$\mu_x = \sum_{\text{All } x} x p(x)$$

Variance and standard deviation of a discrete random variable

$$\sigma_x^2 = \sum_{\text{All } x} (x - \mu_x)^2 p(x) \qquad \sigma_x = \sqrt{\sigma_x^2}$$

Binomial probability formula

$$p(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

Mean, variance, and standard deviation of a binomial random variable

$$\mu_x = np, \ \sigma_x^2 = npq, \text{ and } \sigma_x = \sqrt{npq}$$

Chapter 5 Continuous random variables

The normal probability curve:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Standard normal random variable: $z = \dfrac{x - \mu}{\sigma}$

Chapter 6 Sampling distribution

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\mu_{\bar{p}} = p$$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Chapter 7 Confidence intervals

A z-based confidence interval for a population mean μ with σ known:

Margin of error $E = z_{\alpha/2}\sigma_{\bar{x}} = z_{\alpha/2}\ \sigma/\sqrt{n}$

Interval = $\bar{x} \pm E = \bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n}$

A t-based confidence interval for a population mean μ with σ unknown

$$\left[\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)\right]$$

Confidence interval for the proportion:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Chapter 14 Chi Square test

A goodness of fit test for multinomial probabilities

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - E_i)^2}{E_i}$$

A chi-square test for independence

$$\chi^2 = \sum_{\text{all cells}} \frac{\left(f_{ij} - \hat{E}_{ij}\right)^2}{\hat{E}_{ij}}$$

Chapter 11 Correlation coefficient and simple linear regression analysis

$$s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$= \frac{\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)/n}{n-1}$$

s_x, s_y *are sample standard deviations*

$$SS_{xy} = (n-1)s_{xy}$$

$$SS_{xx} = (n-1)s_x^2 \qquad SS_{yy} = (n-1)s_y^2$$

Simple correlation coefficient

$$r = \frac{s_{xy}}{s_x s_y}$$

Simple linear regression model

$$y = \mu_{y|x} + \varepsilon = \beta_0 + \beta_1 x + \varepsilon$$

Least squares point estimates

$$b_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{s_{xy}}{s_x^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

Sum of squared residuals

$$SSE = SS_{yy} - \frac{SS_{xy}^2}{SS_{xx}}$$

$$= \sum_{i=1}^{n} y_i^2 - b_0 \sum_{i=1}^{n} y_i - b_1 \sum_{i=1}^{n} x_i y_i$$

Standard error

$$s = \sqrt{\frac{SSE}{n-2}}$$

Total variation

$$\sum (y_i - \bar{y})^2 = SS_{yy}$$

Unexplained variation

$$\sum (y_i - \hat{y}_i)^2 = SSE$$

Explained variation

$$\sum (\hat{y}_i - \bar{y})^2$$

Mean square error

$$s^2 = \frac{SSE}{n-2}$$

Simple coefficient of determination

$$r^2 = \frac{\text{explained variation}}{\text{total variation}}$$

F test for the simple linear regression model

$$\frac{\text{explained variation}}{\text{unexplained variation}/(n-2)}$$

(Estimated) Standard error of the estimator b_1

$$\frac{s}{\sqrt{SS_{xx}}}$$

$$\text{Distance value} = \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS_{xx}}$$

(Estimated) Standard error of $\hat{y}$

$$s_{\hat{y}} = s\sqrt{\text{distance value}}$$

Confidence interval for a mean value of y

$$\left[\hat{y} \pm t_{\alpha/2} s\sqrt{\text{distance value}}\right]$$

Prediction interval for an individual value of y

$$\left[\hat{y} \pm t_{\alpha/2} s\sqrt{1 + \text{distance value}}\right]$$

Chapter 12 Multiple regression

The multiple regression model

$$y = \mu_{y|x_1, x_2, \ldots, x_k} + \varepsilon$$

$$= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$$

Standard error

$$s = \sqrt{\frac{SSE}{n-(k+1)}}$$

Mean square error

$$s^2 = \frac{SSE}{n-(k+1)}$$

Multiple coefficient of determination

$$R^2 = \frac{\text{explained variation}}{\text{total variation}}$$

Multiple correlation coefficient $R = \sqrt{R^2}$

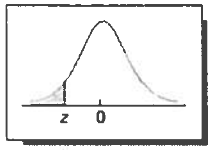
An F test for the linear regression model

$$\frac{\text{explained variation}/k}{\text{unexplained variation}/[n-(k+1)]}$$

Testing the significance of an independent variable

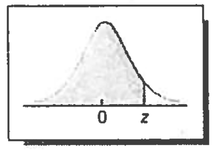
$$t = \frac{b_j}{s_{b_j}}$$

TABLE A.3 **Cumulative Areas under the Standard Normal Curve**



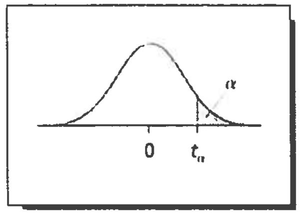
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
−3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
−3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
−3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
−3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
−3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
−3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
−3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
−3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
−3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2482	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

TABLE A.3 **Cumulative Areas under the Standard Normal Curve (*continued*)**



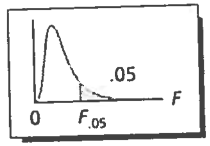
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99897	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

TABLE A.4 **A t Table: Values of t_α for df = 1 through 48**



df	$t_{.100}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
31	1.309	1.696	2.040	2.453	2.744	3.375	3.633
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
33	1.308	1.692	2.035	2.445	2.733	3.356	3.611
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
35	1.306	1.690	2.030	2.438	2.724	3.340	3.591
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
37	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
41	1.303	1.683	2.020	2.421	2.701	3.301	3.544
42	1.302	1.682	2.018	2.418	2.698	3.296	3.538
43	1.302	1.681	2.017	2.416	2.695	3.291	3.532
44	1.301	1.680	2.015	2.414	2.692	3.286	3.526
45	1.301	1.679	2.014	2.412	2.690	3.281	3.520
46	1.300	1.679	2.013	2.410	2.687	3.277	3.515
47	1.300	1.678	2.012	2.408	2.685	3.273	3.510
48	1.299	1.677	2.011	2.407	2.682	3.269	3.505

TABLE A.6 **An *F* Table: Values of $F_{.05}$**

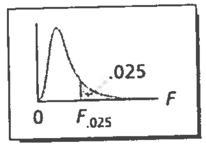


df_2 \ df_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Column heading: Numerator Degrees of Freedom (df_1); row heading: Denominator Degrees of Freedom (df_2)

Source: M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (*F*)-Distribution," *Biometrika* 33 (1943), pp. 73–88. Reproduced by permission of the Biometrika Trustees.

TABLE A.7 An F Table: Values of $F_{.025}$

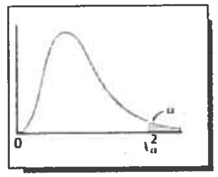


df_2 \ df_1	Numerator Degrees of Freedom (df_1) 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1,001	1,006	1,010	1,014	1,018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

Denominator Degrees of Freedom (df_2)

Source. M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F)-Distribution," *Biometrika* 33 (1943), pp. 73–88. Reproduced by permission of the Biometrika Trustees.

TABLE A.17 **A Chi-Square Table: Values of χ^2_α**



df	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	.0000393	.0001571	.0009821	.0039321	.0157908	2.70554	3.84146	5.02389	6.63490	7.87944
2	.0100251	.0201007	.0506356	.102587	.210720	4.60517	5.99147	7.37776	9.21034	10.5966
3	.0717212	.114832	.215795	.341846	.584375	6.25139	7.81473	9.34840	11.3449	12.8381
4	.206990	.297110	.484419	.710721	.063623	7.77944	9.48773	11.1433	13.2767	14.8602
5	.411740	.554300	.831211	1.145476	1.61031	9.23635	11.0705	12.8325	15.0863	16.7496
6	.675727	.872085	1.237347	1.63539	2.20413	10.6446	12.5916	14.4494	16.8119	18.5476
7	.989265	1.239043	1.68987	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.344419	1.646482	2.17973	2.73264	3.48954	13.3616	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.087912	2.70039	3.32511	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55821	3.24697	3.94030	4.86518	15.9871	18.3070	20.4831	23.2093	25.1882
11	2.60321	3.05347	3.81575	4.57481	5.57779	17.2750	19.6751	21.9200	24.7250	26.7569
12	3.07382	3.57056	4.40379	5.22603	6.30380	18.5494	21.0261	23.3367	26.2170	28.2995
13	3.56503	4.10691	5.00874	5.89186	7.04150	19.8119	22.3621	24.7356	27.6883	29.8194
14	4.07468	4.66043	5.62872	6.57063	7.78953	21.0642	23.6848	26.1190	29.1413	31.3193
15	4.60094	5.22935	6.26214	7.26094	8.54675	22.3072	24.9958	27.4884	30.5779	32.8013
16	5.14224	5.81221	6.90766	7.96164	9.31223	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.69724	6.40776	7.56418	8.67176	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.26481	7.01491	8.23075	9.39046	10.8649	25.9894	28.8693	31.5264	34.8053	37.1564
19	6.84398	7.63273	8.90655	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5822
20	7.43386	8.26040	9.59083	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.03366	8.89720	10.28293	11.5913	13.2396	29.6151	32.6705	35.4789	38.9321	41.4010
22	8.64272	9.54249	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7956
23	9.26042	10.19567	11.6885	13.0905	14.8479	32.0069	35.1725	38.0757	41.6384	44.1813
24	9.88623	10.8564	12.4011	13.8484	15.6587	33.1963	36.4151	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9278
26	11.1603	12.1981	13.8439	15.3791	17.2919	35.5631	38.8852	41.9232	45.6417	48.2899
27	11.8076	12.8786	14.5733	16.1513	18.1138	36.7412	40.1133	43.1944	46.9630	49.6449
28	12.4613	13.5648	15.3079	16.9279	18.9392	37.9159	41.3372	44.4607	48.2782	50.9933
29	13.1211	14.2565	16.0471	17.7083	19.7677	39.0875	42.5569	45.7222	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4926	20.5992	40.2560	43.7729	46.9792	50.8922	53.6720
40	20.7065	22.1643	24.4331	26.5093	29.0505	51.8050	55.7585	59.3417	63.6907	66.7659
50	27.9907	29.7067	32.3574	34.7642	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.5346	37.4848	40.4817	43.1879	46.4589	74.3970	79.0819	83.2976	88.3794	91.9517
70	43.2752	45.4418	48.7576	51.7393	55.3290	85.5271	90.5312	95.0231	100.425	104.215
80	51.1720	53.5400	57.1532	60.3915	64.2778	96.5782	101.879	106.629	112.329	116.321
90	59.1963	61.7541	65.6466	69.1260	73.2912	107.565	113.145	118.136	124.116	128.299
100	67.3276	70.0648	74.2219	77.9295	82.3581	118.498	124.342	129.561	135.807	140.169

Source: C. M. Thompson, "Tables of the Percentage Points of the χ^2 Distribution," *Biometrika* 32 (1941), pp. 188–89. Reproduced by permission of the Biometrika Trustees.