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LESSON 8
HYPOTHESIS TESTING

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LESSON 6 IN A NUTSHELL

Normal Probability Distribution

<u>The sample size has been specified.</u>

SAMPLING DISTRIBUTION OF $\bar{x}$

If it is <u>NOT NORMALLY DISTRIBUTED</u>,

with a large enough sample n > 30, **"Central Limit Theorem"**, it will approximately be NORMALLY DISTRIBUTED.

SAMPLING DISTRIBUTION OF $\bar{p}$

$$np \geq 5 \qquad n(1-p) \geq 5$$

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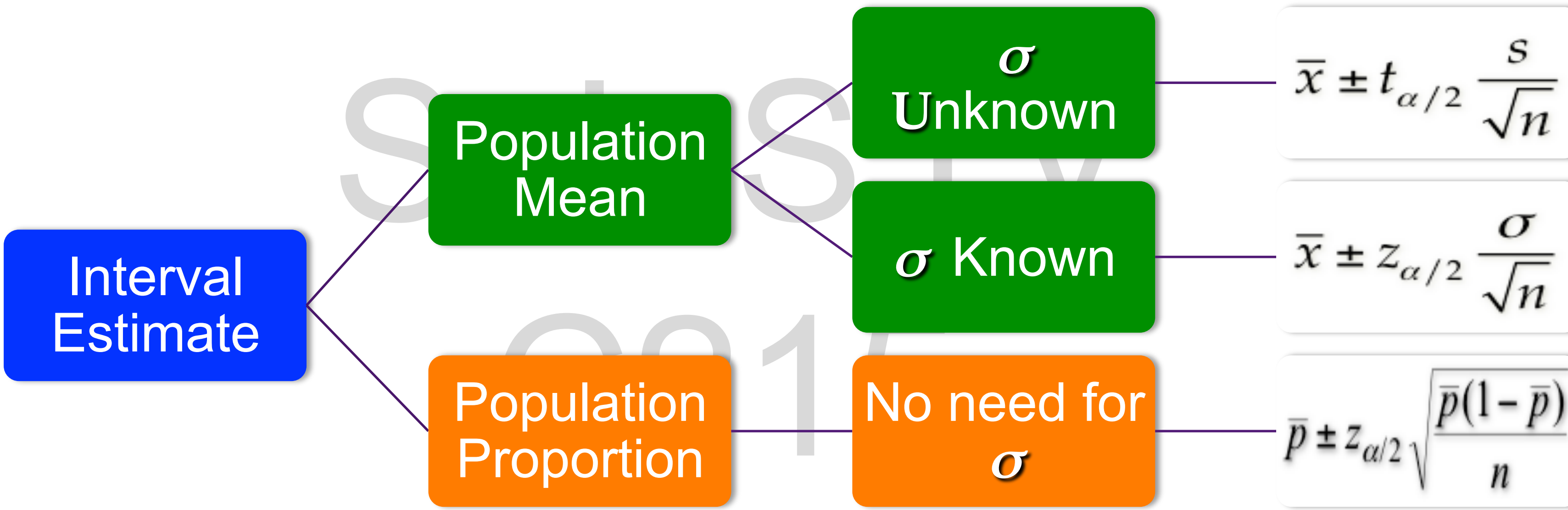
LESSON 7 IN A NUTSHELL



LESSON 7 IN A NUTSHELL

- Sample Size
  - Population Mean
    - Margin of Error — $E = z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$
    - $n = \dfrac{(z_{\alpha/2})^2\sigma^2}{E^2}$
  - Population Proportion
    - Margin of Error — $E = z_{\alpha/2}\sqrt{\dfrac{\overline{p}(1-\overline{p})}{n}}$
    - $n = \dfrac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$

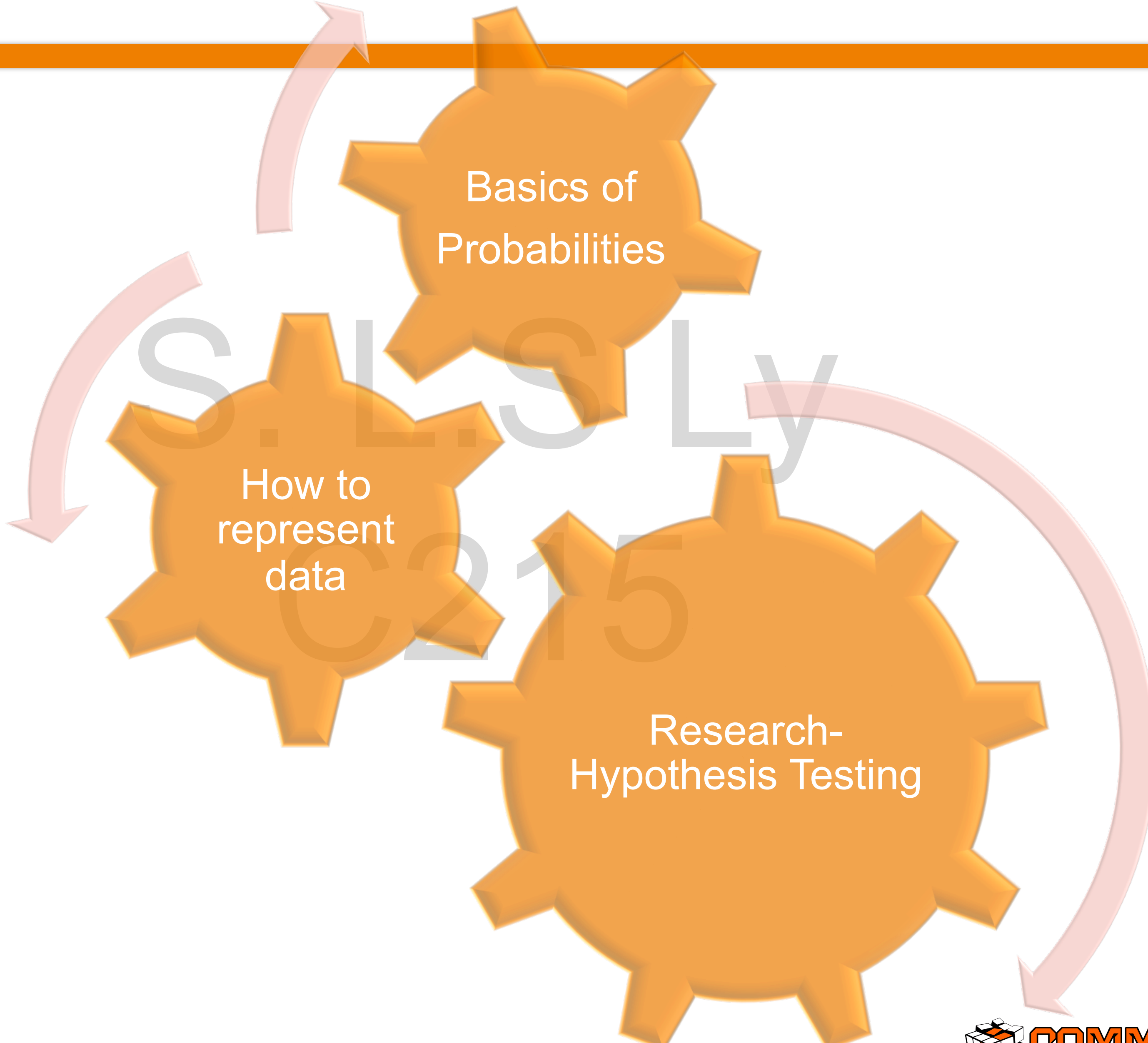
REST OF THE SEMESTER

Simple Linear Regression

Goodness of Fit and Independence Test

Multiple Linear Regression

Hypothesis Testing


Basics of Probabilities
How to represent data
Research-Hypothesis Testing

The Null and Alternative Hypotheses and Errors in Hypothesis Testing

Type I and Type II Errors and Their Probabilities

z Tests about a population Mean (σ unknown) One-Sided Alternatives

z Tests about a population Mean (σ known)Two-Sided Alternatives

t Tests about Population Mean (σ unknown)

z Tests about Population Proportion

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7 STEPS TO HYPOTHESIS TESTING

Step 1-**Set up Hypotheses**

- What is *α?* Is it a one-tail test? or a two tail test?
- Set up null hypothesis H_0, Set up alternative hypothesis H_a

Step 2- **What is the appropriate test statistic to use?**

Step 3- **Calculate the test statistics value**

Step 4- **Find the critical value for the test statistic.**

Step 5- **Define your decision rule**

Step 6- **Make your decision**

Step 7-**Interpret the conclusion in context**

DEVELOPING HYPOTHESES

Hypothesis testing Introduction

A student and a teacher are discussing about the number of students attending class after the midterm.

The teacher says that from the past 60% of students attend class after the midterm.

The students sees today's class differently, she sees about 70% attending.

In order to use statistics to test whether the student is right, or the teacher's information is correct, we have to set up hypotheses.

$$H_0: \mu \geq \mu_0$$
$$H_a: \mu < \mu_0$$

One-tailed (lower-tail)

$$H_0: \mu \leq \mu_0$$
$$H_a: \mu > \mu_0$$

One-tailed (upper-tail)

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

Two-tailed

Bowerman, et al. (2017) pp. 386

There are TWO Hypotheses.

Null Hypothesis

Ho : The defendant is innocent

Alternative/Research Hypothesis

Ha : The defendant is guilty

Null Hypothesis

Ho : The defendant is innocent

Convicting the defendant→ Rejection Ho.

There is enough evidence to prove that the defendant is NOT innocent.

Alternative/Research Hypothesis

Ha : The defendant is guilty

Acquitting the defendant→ do not reject Ho.

There is not enough evidence to prove that the defendant is guilty.

NULL HYPOTHESIS H_o

- You want to **challenge** the null hypothesis.
- The null hypothesis is a statement about the value of a population parameter, **from the past**.
- It is **assumed to be true** unless…

  we have evidence to the contrary.

The null hypothesis must always contain the equal sign.

ALTERNATIVE HYPOTHESIS H_A

- The alternative hypothesis is

the research hypothesis,

- opposite of what is stated by the null hypothesis.

- **Type I and Type II Errors and Their Probabilities**
- **z Tests about a population Mean (σ unknown) One-Sided Alternatives**
- **z Tests about a population Mean (σ known)Two-Sided Alternatives**
- **t Tests about Population Mean (σ unknown)**
- **z Tests about Population Proportion**

TYPE I & TYPE II

Conclusion	Population Condition: H_0 True ($\mu \leq 12$)	Population Condition: H_0 False ($\mu > 12$)
Accept H_0 (Conclude $\mu \leq 12$)	Correct Decision	Type II Error
Reject H_0 (Conclude $\mu > 12$)	Type I Error	Correct Decision

Bowerman, et al. (2017) pp. 387

TYPE I

When you Reject H_o

But in fact H_o is true

Type I error is made when an innocent person is wrongly convicted.

To say Accept Ho, it is technically incorrect.

TYPE II

When you DO NOT REJECT H_o

But in fact, H_o is actually FALSE

Type II error occurs when a guilty person is acquitted.

PROBLEM # 8.2

Following a major earthquake, the site engineer must determine whether the stadium is structurally sound for an upcoming athletic event. If the null hypothesis is “ the stadium is structurally sound,” and the alternative hypothesis is “ the stadium is not structurally sound,” which type of error (Type I or Type II) would the engineer least like to commit?

The engineer would least like to commit a Type II error since a lot of people could be killed if this occurred. A Type II error would be committed if he decided that the stadium was structurally sound when it was not.

PROBLEM # 8.3

A researcher wants to carry out a hypothesis test involving the mean for a sample of size n= 18. She does not know the true value of the population standard deviation, but is reasonably sure that the underlying population is approximately normally distributed. Should she use a z-test or a t-test in carrying out the analysis? Why?

We should use a t-test to carry out the analysis since σ is unknown but we are reasonably sure the population is approximately normally distributed.

PROBLEM #8.5

It has been claimed that no more than 5% of the units coming off an assembly line are defective. Formulate a null hypothesis and an alternative hypothesis for this situation. Will the test be one-tail or two-tail? Why? If the test is one-tail, will it be left-tail or right-tail? Why?

Let p_0 = population proportion of defective units.

H_0: $p_0 \leq 0.05$ H_1: $p_0 > 0.05$

This is a one-tail test since this is a directional claim ("no more than 5%"). It is a right-tail test since the alternative hypothesis has a greater-than sign. The rejection region is located in the right tail of the standard normal curve.

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POPULATION MEAN: σ KNOWN

One-tailed Test

Two-tailed Test

Relationship between Interval Estimation and Hypothesis Testing

Bowerman, et al. (2017) pp. 391

7 STEPS TO HYPOTHESIS TESTING

Step 1-**Set up Hypotheses**

- What is *α?* Is it a one-tail test? or a two tail test?
- Set up null hypothesis H_0, Set up alternative hypothesis H_a

Step 2- **What is the appropriate test statistic to use?**

Step 3- **Calculate the test statistics value**

Step 4- **Find the critical value for the test statistic.**

Step 5- **Define your decision rule**

Step 6- **Make your decision**

Step 7-**Interpret the conclusion in context**

TWO APPROCHES

P-value approach

Critical value approach

PROBLEM # 8.9

For a sample of 35 items from a population for which the standard deviation is σ = 20.5, the sample mean is 458.0. At the 0.05 level of significance, test H_0: μ = 450 versus H_1: $\mu \neq$ 450. Determine and interpret the p-value for the test.

Step 1- Set up Hypotheses

H_0: μ = 450 H_1: $\mu \neq$ 450

Level of significance: α = 0.05 , $\alpha/2$ = 0.025 –two tailed test

Step 2- What is the appropriate test statistic to use?

$\bar{X}$ = 458, n = 35 (known: σ = 20.5) **,** use the z-test

PROBLEM # 8.9

For a sample of 35 items from a population for which the standard deviation is σ = 20.5, the sample mean is 458.0. At the 0.05 level of significance, test H_0: μ = 450 versus H_1: $\mu \neq$ 450. Determine and interpret the p-value for the test.

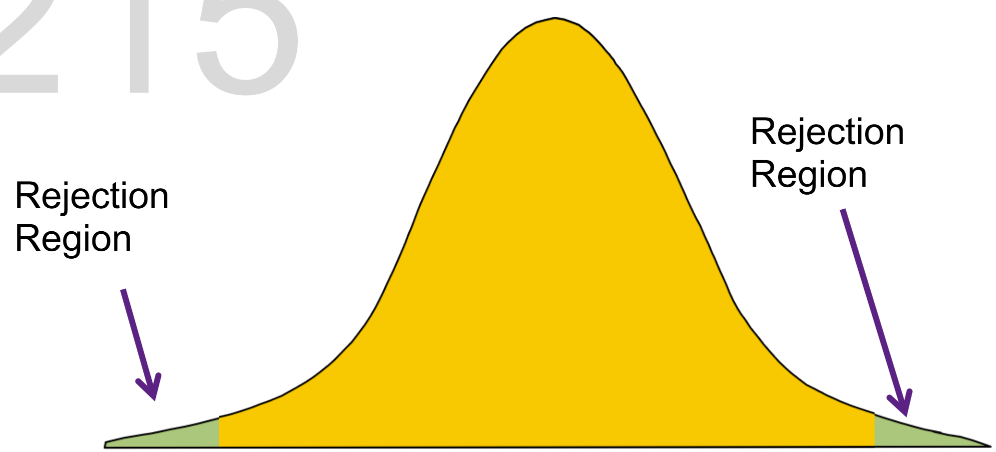
Step 3-Calculate the test statistics value

Find z_{observed}

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{458 - 450}{20.5/\sqrt{35}} = 2.31$$

Step 4- Find the critical value for the test statistic.

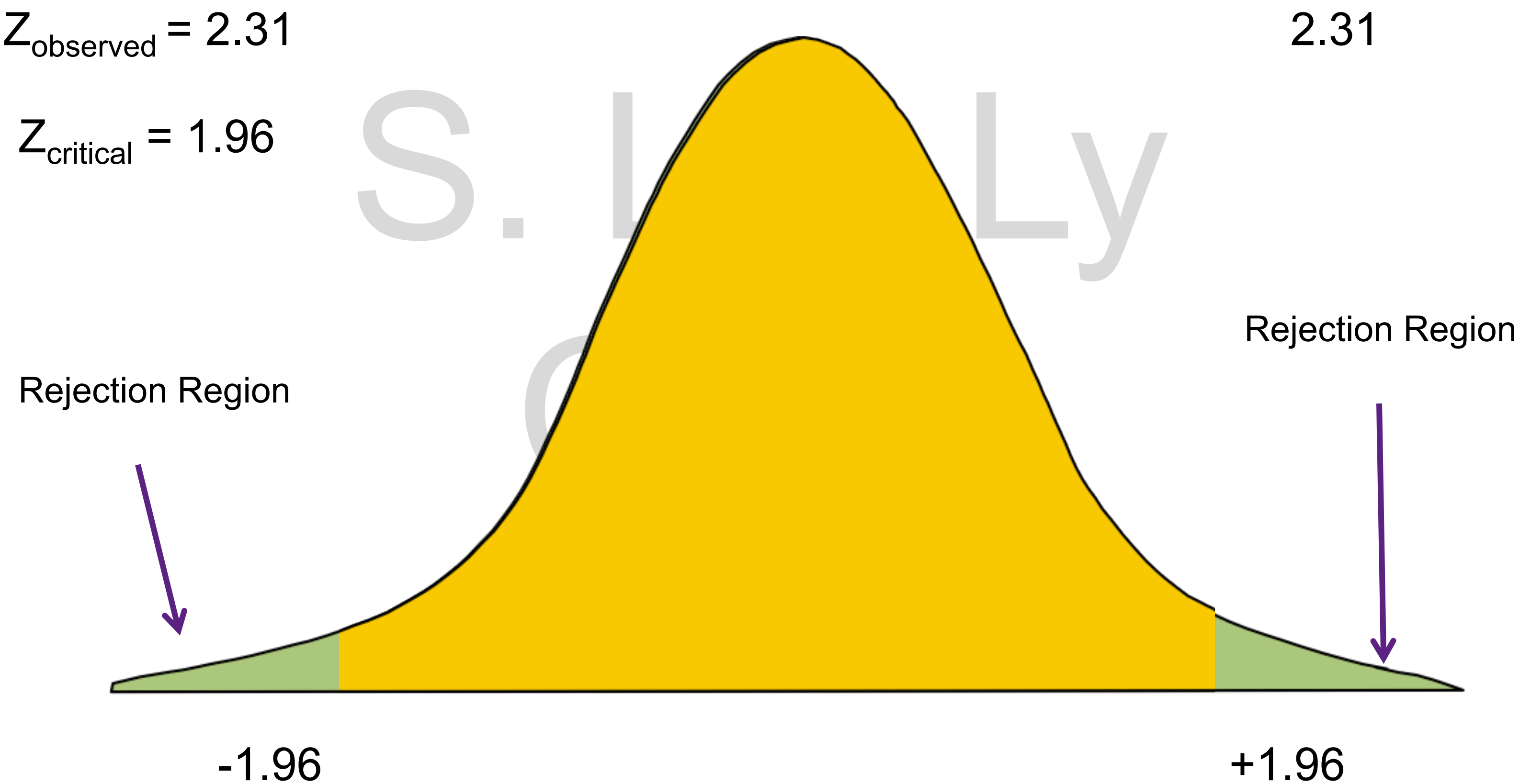
z_{critical} = 1.96 and z = -1.96



$Z_{observed}$ falls in the rejection region, therefore Reject H_o

$Z_{observed} = 2.31$

$Z_{critical} = 1.96$



PROBLEM # 8.9

For a sample of 35 items from a population for which the standard deviation is $\sigma = 20.5$, the sample mean is 458.0. At the 0.05 level of significance, test H_0: $\mu = 450$ versus H_1: $\mu \neq 450$. Determine and interpret the p-value for the test.

Step 5- Define your decision rule

Reject H_0 if the calculated $z < -1.96$ or > 1.96, otherwise do not reject.

Step 6- Make your decision

Since calculated test statistic falls in rejection region ($z = 2.31 > 1.96$), reject H_0

PROBLEM # 8.9

For a sample of 35 items from a population for which the standard deviation is $\sigma = 20.5$, the sample mean is 458.0. At the 0.05 level of significance, test H_0: $\mu = 450$ versus H_1: $\mu \neq 450$. Determine and interpret the p-value for the test.

Step 7- Interpret the conclusion in context

At the 0.05 level, the results suggest that the population mean is not 450.

P-Value approach

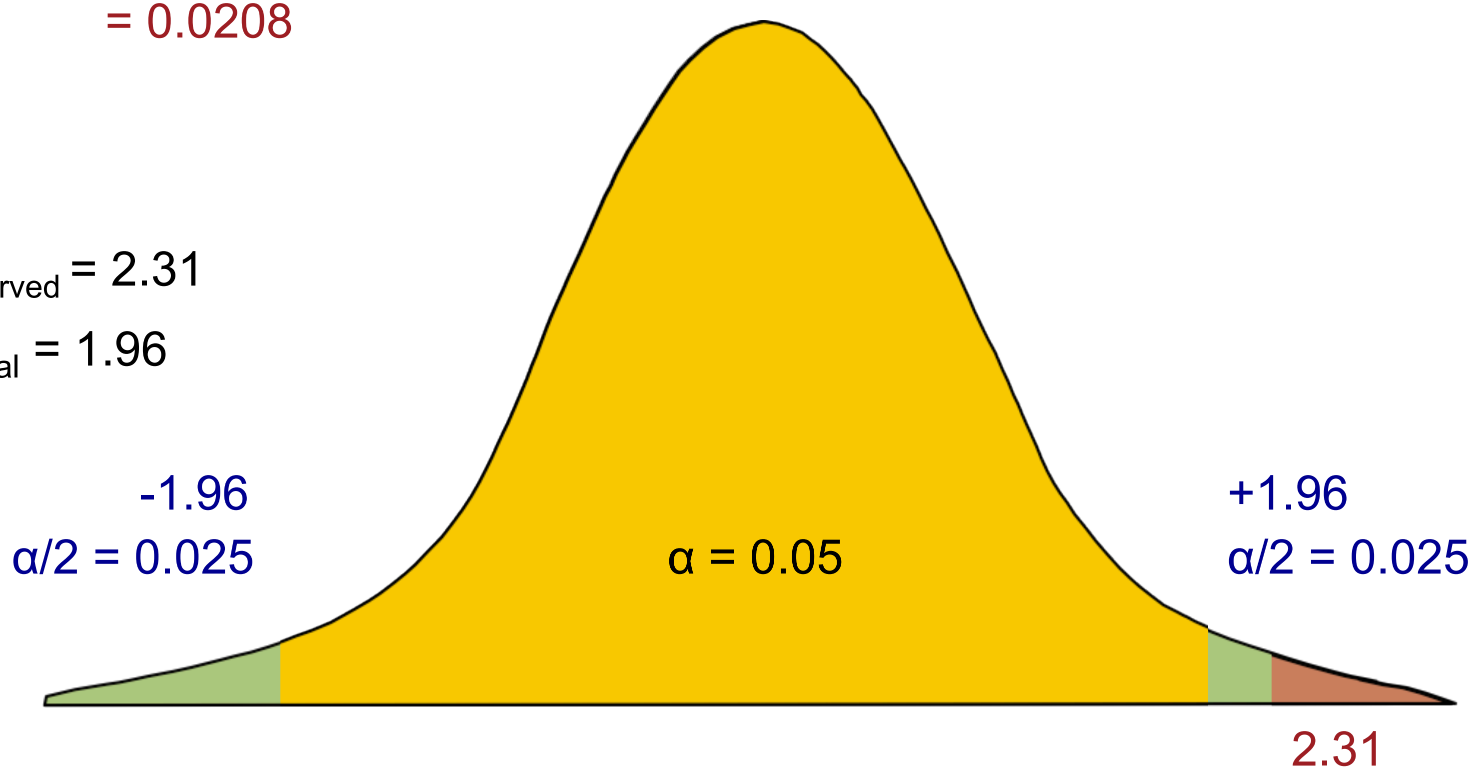
What is the probability of  ? P() = 0.0104

p-value = 0.0104 + 0.0104
= 0.0208

α = 0.025 + 0.025 = 0.05

$Z_{observed}$ = 2.31

$Z_{critical}$ = 1.96



P-VALUE APPROACH

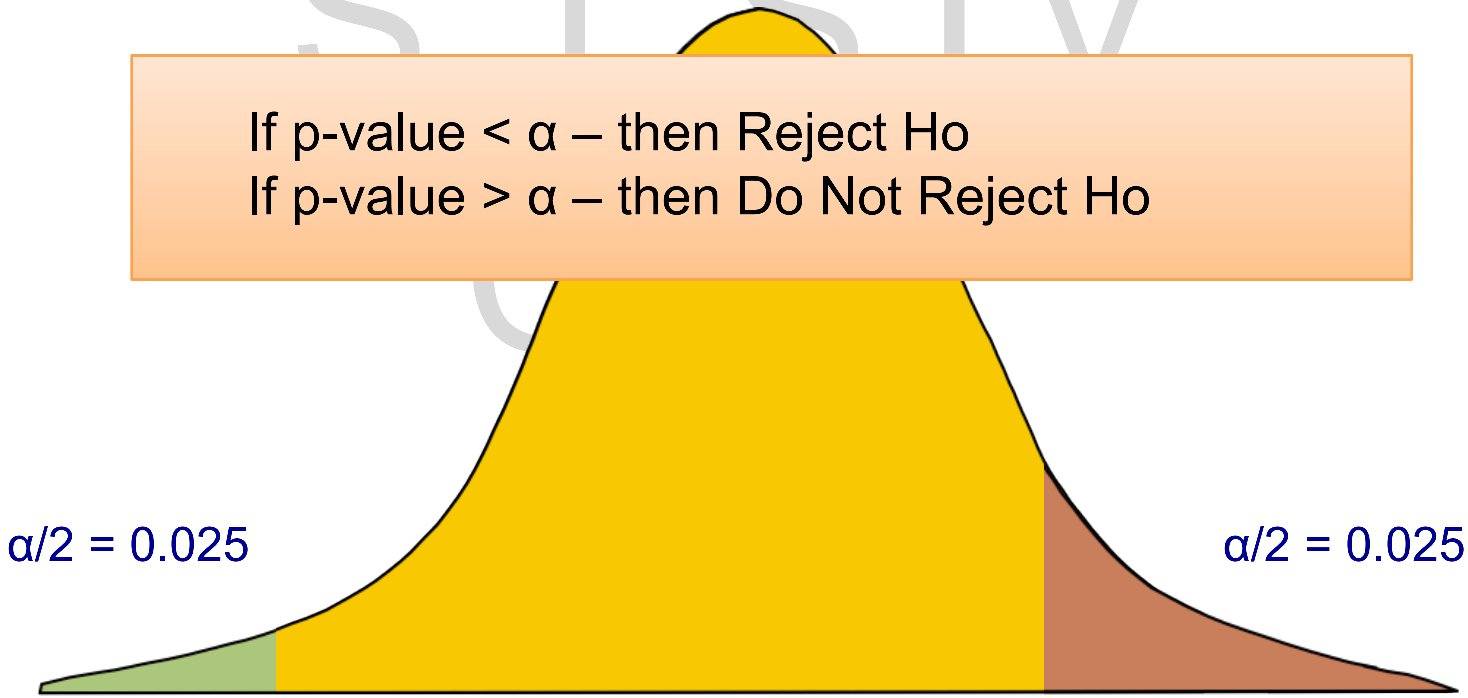
p-value = 0.0104 + 0.0104
= 0.0208

α = 0.025 + 0.025 = 0.05

DO NOT REJECT Ho

REJECT Ho

If p-value < α – then Reject Ho
If p-value > α – then Do Not Reject Ho



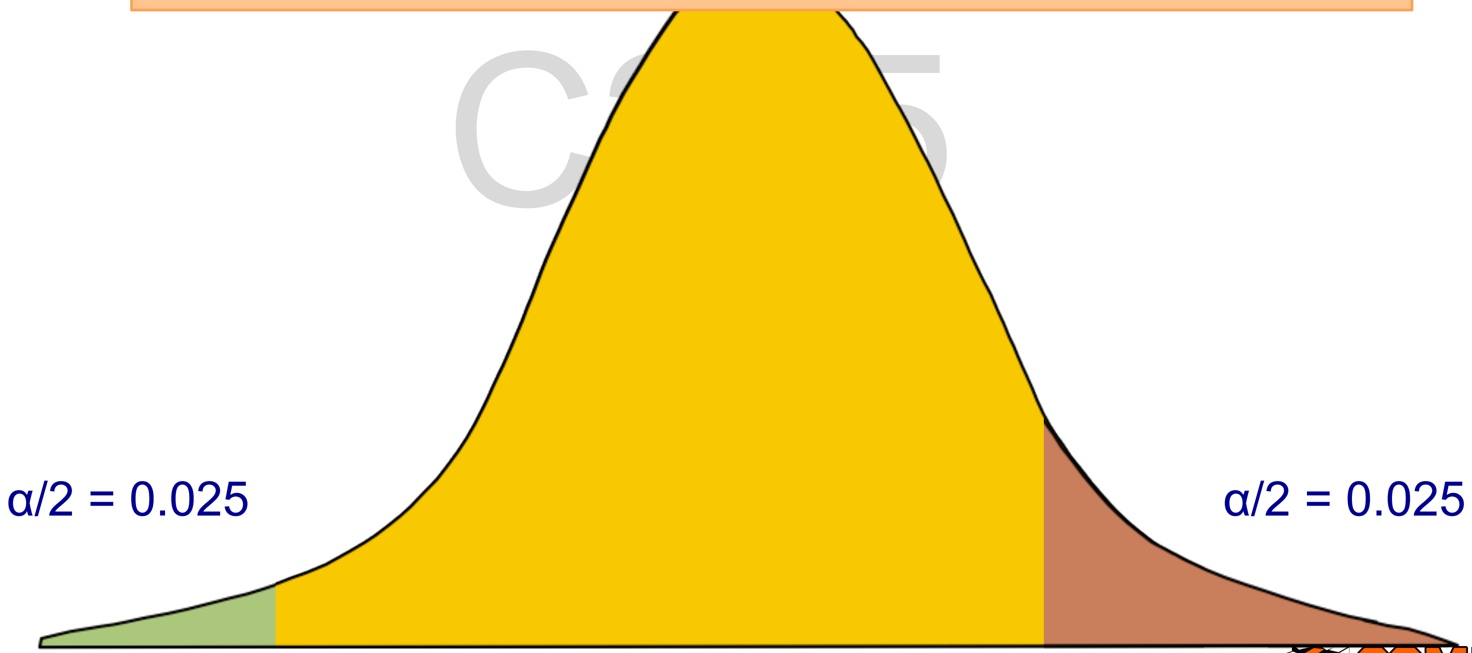
Bowerman, et al. (2017) pp. 397

P-VALUE APPROACH

p-value = 0.0104 + 0.0104
= 0.0208

α = 0.025 + 0.025 = 0.05

If p-value < α – then Reject Ho
If p-value > α – then Do Not Reject Ho



PROBLEM #8.10

For each of the following tests and z-values, determine the p-value for the test:

a. Right-tail test and z=1.54

p-value =P(z ≥1.54) = 1.0000 - 0.9382 = 0.0618

b. Left-tail test and z=-1.03

p-value =P(z ≤ -1.03) = 0.1515

c. Two-tail test and z=1.27

p-value =2P(z ≤ -1.27) = 2(0.102) = 0.204

PROBLEM # 8.11

For a sample of 12 items from a normally distributed population for which the standard deviation is σ=17.0, the sample mean is 230.8. At the 0.05 level of significance, test H_0: $\mu \leq 220$ versus H_1: $\mu > 220$. Determine and interpret the p-value for the test.

Step 1- Set up Hypotheses

H_0: $\mu \leq 220$ H_1: $\mu > 220$

Level of significance: $\alpha = 0.05$, one tail test

Step 2- What is the appropriate test statistic to use?

$\overline{X} = 230.8$, n = 12 (known: $\sigma = 17$ and the population is normally distributed.)

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PROBLEM # 8.11

For a sample of 12 items from a normally distributed population for which the standard deviation is σ =17.0, the sample mean is 230.8. At the 0.05 level of significance, test H_0: $\mu \leq 220$ versus H_1: $\mu > 220$. Determine and interpret the p-value for the test.

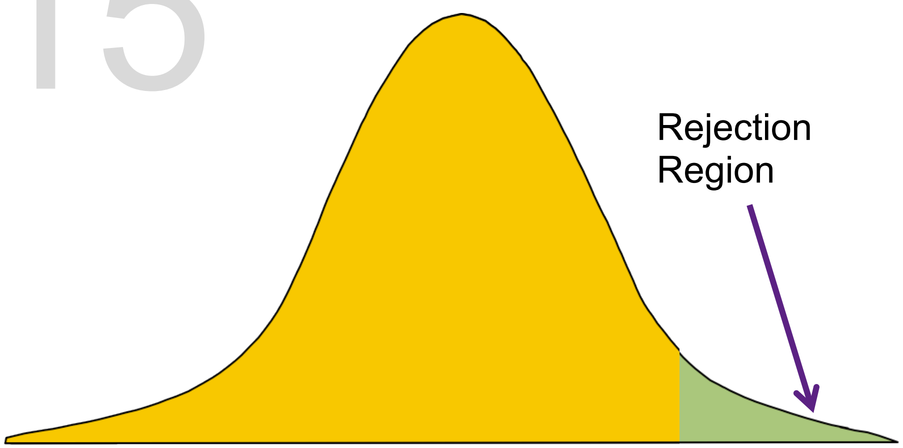
Step 3-Calculate the test statistics value

Find z_{observed}

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{230.8 - 220}{17/\sqrt{12}} = 2.20$$

Step 4- Find the critical value for the test statistic.

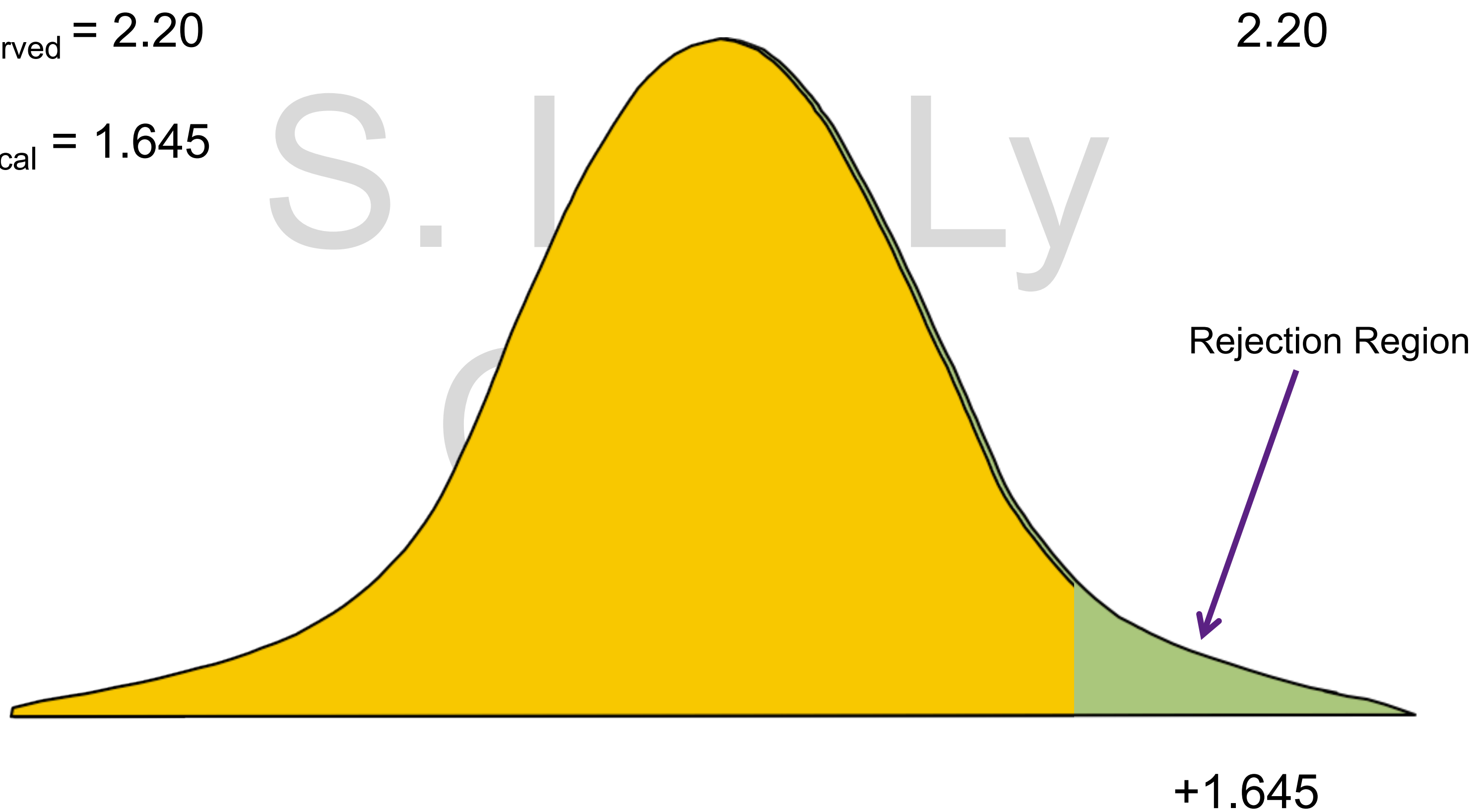
$z_{\text{critical}} = 1.645$



$Z_{observed}$ falls in the rejection region, therefore Reject H_o

$Z_{observed} = 2.20$

$Z_{critical} = 1.645$



PROBLEM # 8.11

For a sample of 12 items from a normally distributed population for which the standard deviation is σ =17.0, the sample mean is 230.8. At the 0.05 level of significance, test H_0: $\mu \leq 220$ versus H_1: $\mu > 220$. Determine and interpret the p-value for the test.

Step 5- Define your decision rule

Reject H_0 if the calculated $z > 1.645$, otherwise do not reject.

Step 6- Make your decision

Since calculated test statistic falls in rejection region ($z = 2.20 > 1.645$), reject H_0.

PROBLEM # 8.11

For a sample of 12 items from a normally distributed population for which the standard deviation is σ =17.0, the sample mean is 230.8. At the 0.05 level of significance, test H_0: $\mu \leq 220$ versus H_1: $\mu > 220$. Determine and interpret the p-value for the test.

Step 7- Interpret the conclusion in context

At the 0.05 level, the results suggest that the population mean is greater than 220.

P-Value approach

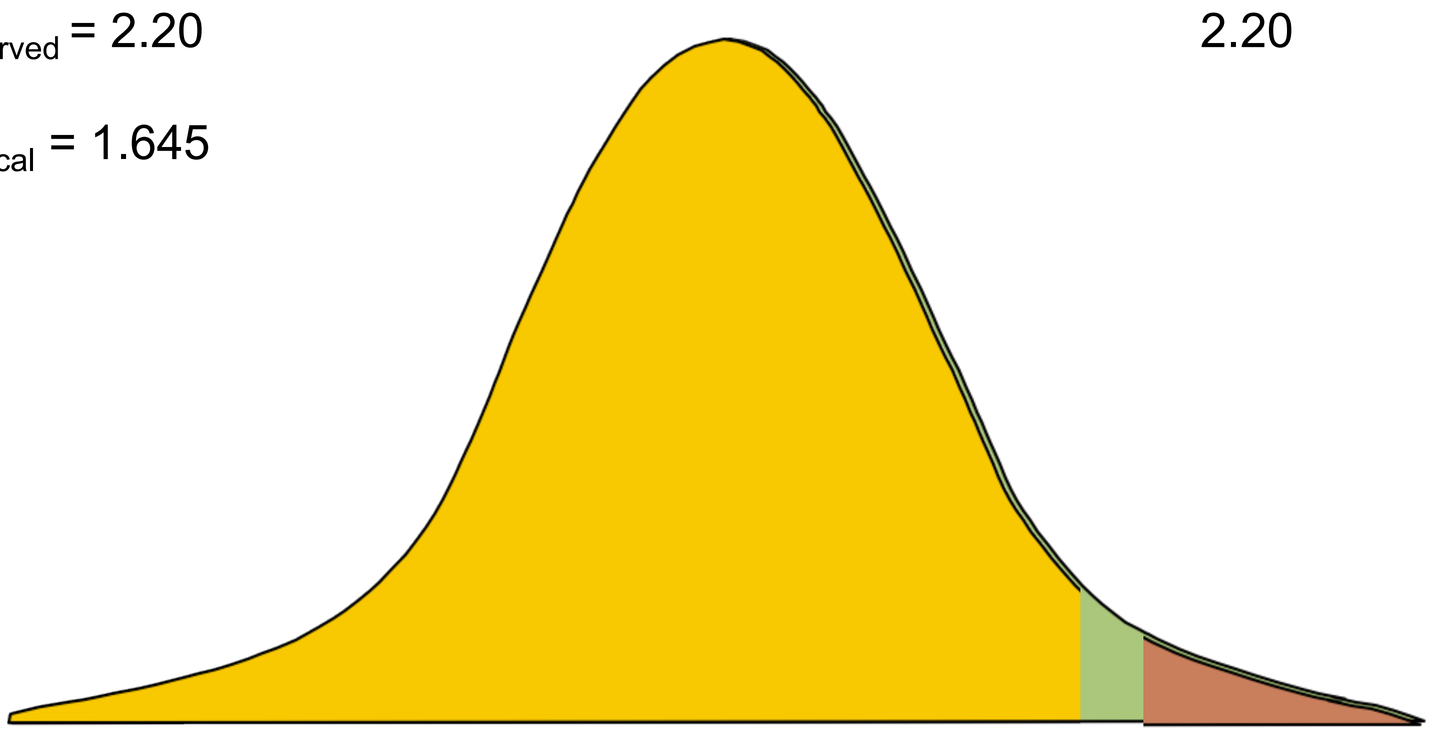
p-value = 0.0139

Since p-value (0.0139) < α = 0.05 , then Reject Ho

What is the probability of  ? P() = 0.0139

$Z_{observed}$ = 2.20

$Z_{critical}$ = 1.645



PROBLEM # 8.13

Based on the sample data, a confidence interval has been constructed such that we have 90% confidence that the population mean is between 120 and 180. Given this information, provide the conclusion that would be reached for each of the following hypothesis tests at the α=0.10 level:

a. H_0: μ = 170 versus H_1: $\mu \neq$ 170

Do not reject H_0 since 170 is in the 90% confidence interval given.

b. H_0: μ = 110 versus H_1: $\mu \neq$ 110

Reject H_0 since 110 is not in the 90% confidence interval given.

POPULATION MEAN: σ UNKNOWN

One-tailed Test

Two-tailed Test

Bowerman, et al. (2017) pp. 402

PROBLEM # 8.16

For a simple random sample of 15 items from a population that is approximately normally distributed, x =82.0 and s=20.5. At the 0.05 level of significance, test H_0: $\mu \geq 90.0$ versus H_1: $\mu < 90.0$.

Step 1- Set up Hypotheses

H_0: $\mu \geq 90.0$ $\qquad$ H_1: $\mu < 90.0$

Level of significance: $\alpha = 0.05$, one tail test

Step 2- What is the appropriate test statistic to use?

$\bar{X}$ = 82.0, s = 20.5, n = 15 (Note: population is approximately normally distributed.)

PROBLEM # 8.16

For a simple random sample of 15 items from a population that is approximately normally distributed, $\bar{x}$ =82.0 and s=20.5. At the 0.05 level of significance, test H_0: $\mu \geq 90.0$ versus H_1: $\mu < 90.0$.

Step 3-Calculate the test statistics value

Find $t_{observed}$ $$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{82.0 - 90.0}{20.5/\sqrt{15}} = -1.511$$

Step 4- Find the critical value for the test statistic.

$\alpha = 0.05$

d.f. = (n - 1) = (15 - 1) = 14

$t_{critical}$ = -1.761

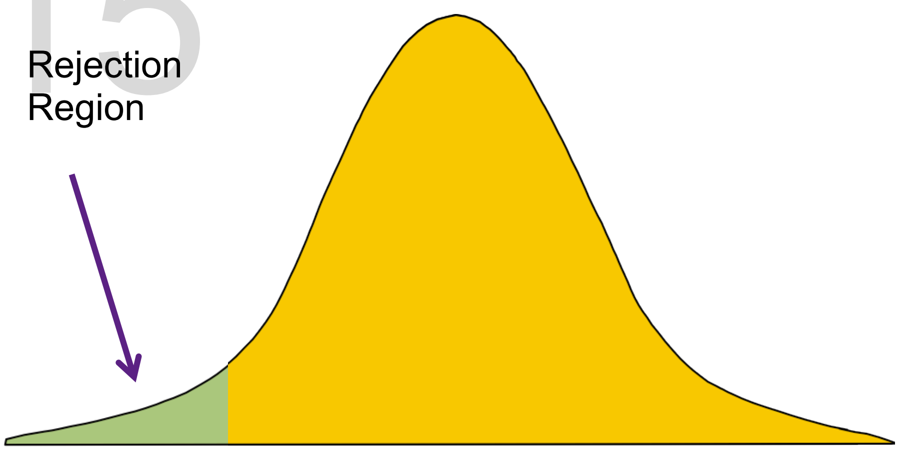
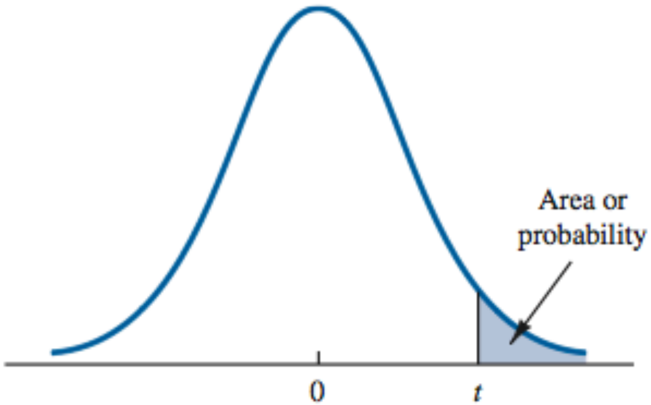


TABLE 2 t DISTRIBUTION



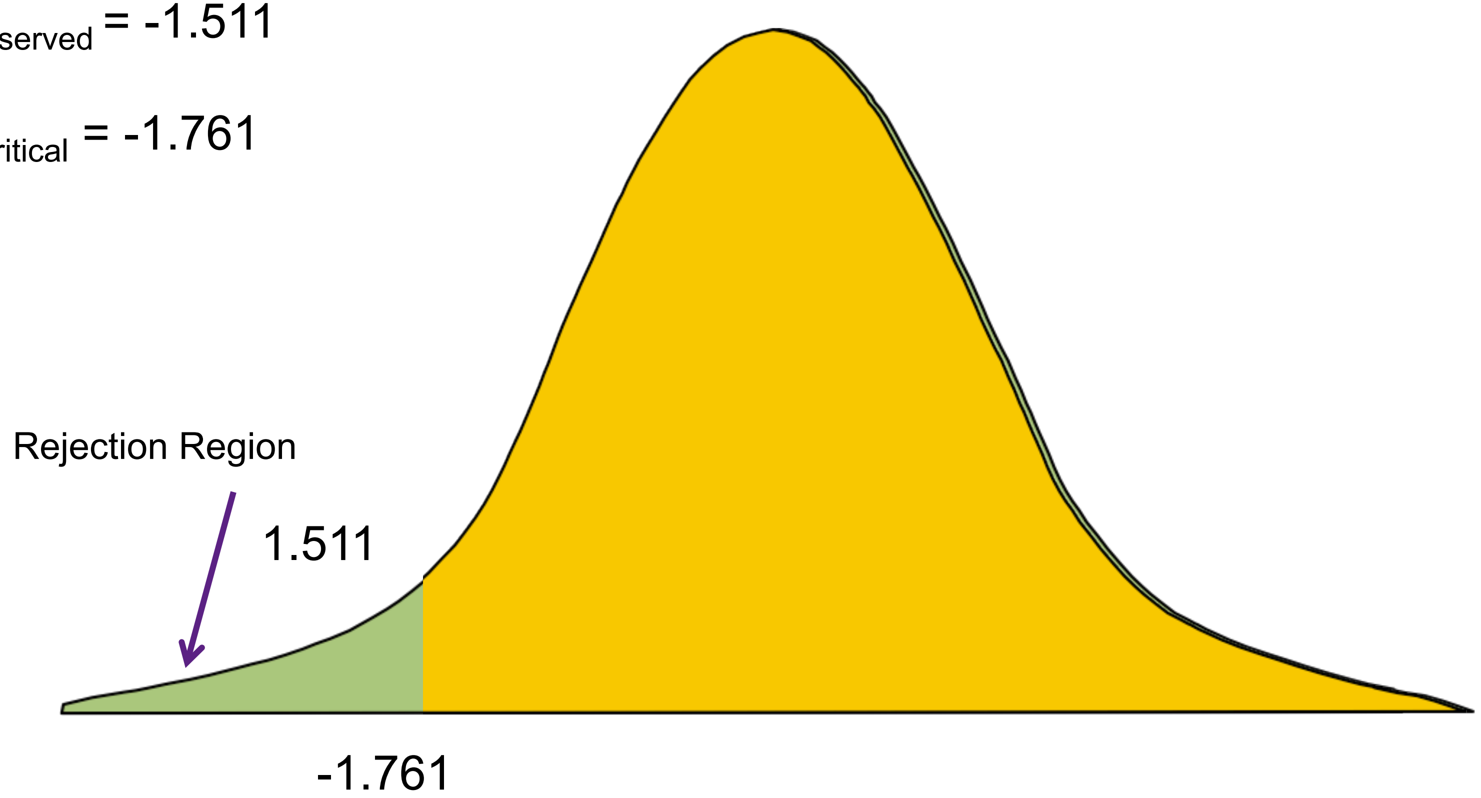
Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921

$t_{observed}$ falls in the rejection region, therefore Reject H_o

$t_{observed} = -1.511$

$t_{critical} = -1.761$



PROBLEM # 8.16

For a simple random sample of 15 items from a population that is approximately normally distributed, x =82.0 and s=20.5. At the 0.05 level of significance, test H_0: $\mu \geq 90.0$ versus H_1: $\mu < 90.0$.

Step 5- Define your decision rule

Reject H_0 if the calculated t < -1.761, otherwise do not reject.

Step 6- Make your decision

Calculated test statistic falls in nonrejection region, do not reject H_0.

PROBLEM # 8.16

For a simple random sample of 15 items from a population that is approximately normally distributed, x =82.0 and s=20.5. At the 0.05 level of significance, test H_0: $\mu \geq 90.0$ versus H_1: $\mu < 90.0$.

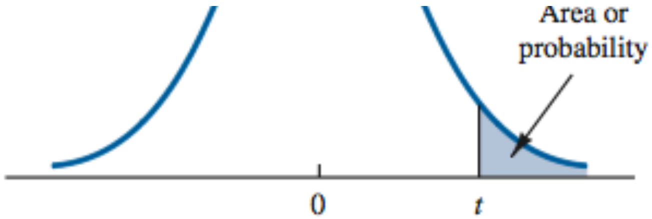
Step 7- Interpret the conclusion in context

At the 0.05 level, the results suggest that the population mean could be at least 90.0.

P-Value approach

p-value = ?

$t_{observed}$ = -1.511



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Can you find -1.511 in the table to find its probability?

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921

P-value of -1.511 is between 0.05 and 0.10

Somewhere here!

PROBLEM # 8.16

For a simple random sample of 15 items from a population that is approximately normally distributed, x =82.0 and s=20.5. At the 0.05 level of significance, test H_0: $\mu \geq 90.0$ versus H_1: $\mu < 90.0$.

Step 7- Interpret the conclusion in context

At the 0.05 level, the results suggest that the population mean could be at least 90.0.

P-Value approach

p-value of -1.511 is between 0.05 and 0.10

p-value is definitely larger than α= 0.05

Therefore, do not reject Ho.

PROBLEM # 8.17

The International Coffee Association has reported the mean daily coffee consumption for U.S. residents as 1.65 cups. Assume that a sample of 38 people from a North Carolina city consumed a mean of 1.84 cups of coffee per day, with a standard deviation of 0.85 cups. In a two-tail test at the 0.05 level, could the residents of this city be said to be significantly different from their counterparts across the nation? *Source: coffeeresearch.org, August 8, 2006.*

Step 1- Set up Hypotheses

H_0: $\mu = 1.65$ (the mean daily coffee consumption in this city is the same as for all U.S. residents)

H_1: $\mu \neq 1.65$ (the mean daily coffee consumption in this city differs from the overall U.S.)

Level of significance: $\alpha = 0.05$, two tail test

Step 2- What is the appropriate test statistic to use?

$\overline{X} = 1.84$, s = 0.85, n = 38

PROBLEM # 8.17

The International Coffee Association has reported the mean daily coffee consumption for U.S. residents as 1.65 cups. Assume that a sample of 38 people from a North Carolina city consumed a mean of 1.84 cups of coffee per day, with a standard deviation of 0.85 cups. In a two-tail test at the 0.05 level, could the residents of this city be said to be significantly different from their counterparts across the nation? *Source: coffeeresearch.org, August 8, 2006.*

Step 3-Calculate the test statistics value

Find t_{observed}

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{1.84 - 1.65}{0.85 / \sqrt{38}} = 1.378$$

Step 4- Find the critical value for the test statistic.

$\alpha = 0.05$

d.f. = (n - 1) = (38 - 1) = 37

t_{critical} = -2.026 and 2.026

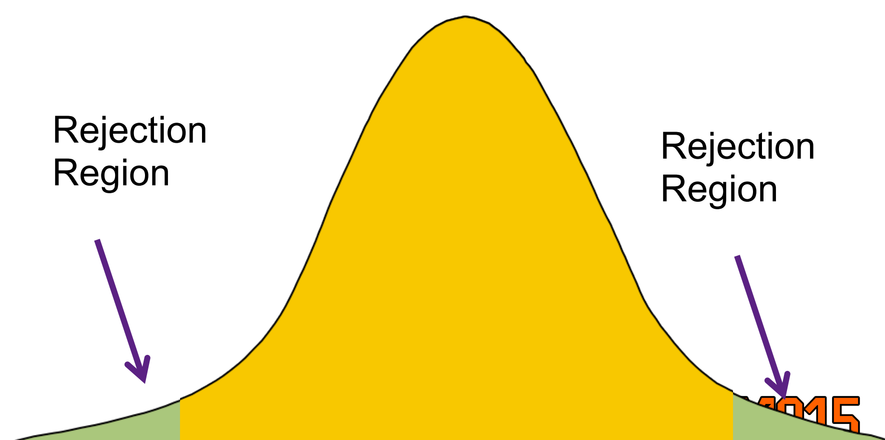


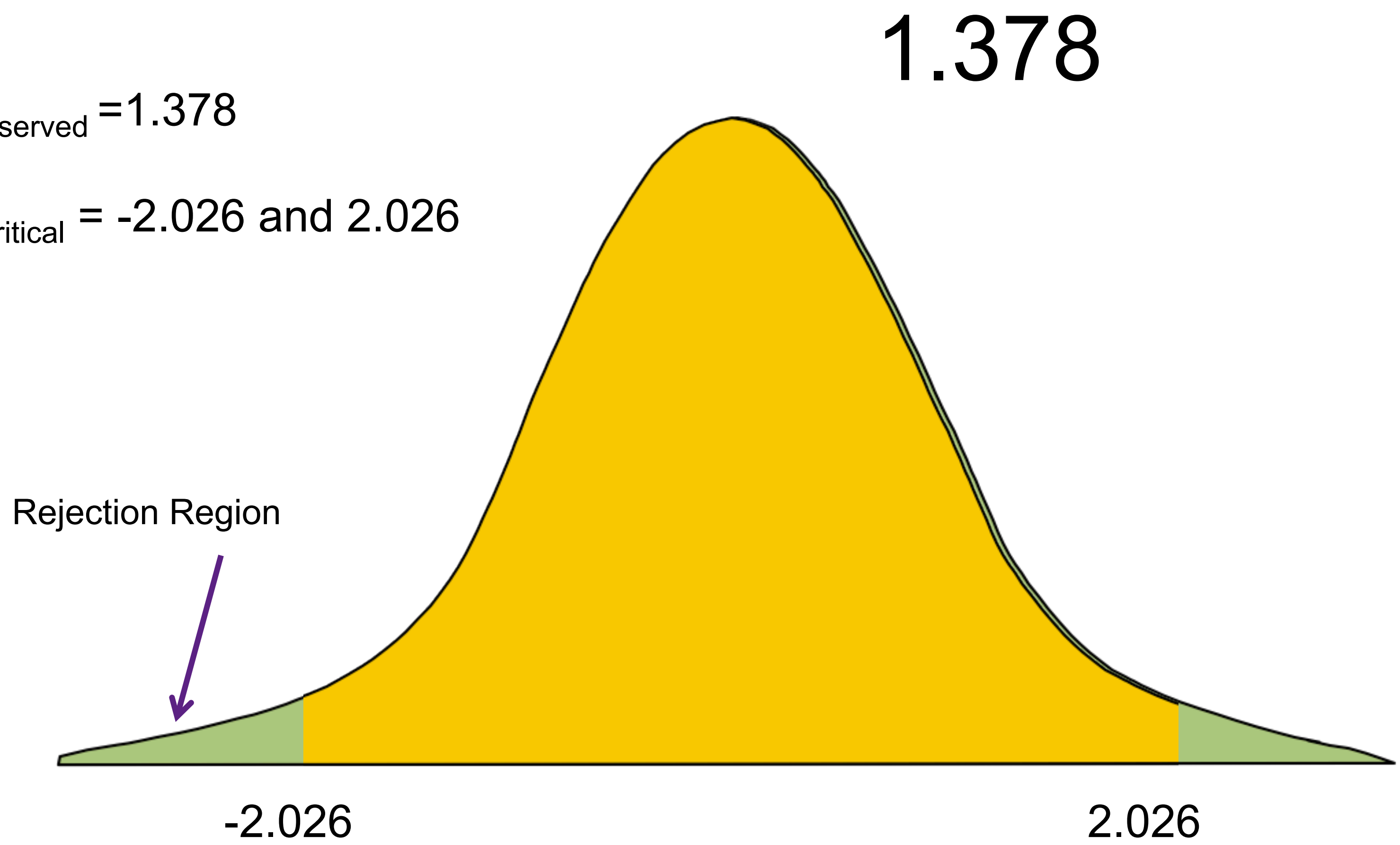
TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690

$t_{observed}$ falls in the rejection region, therefore Reject H_o

$t_{observed}$ =1.378

$t_{critical}$ = -2.026 and 2.026



PROBLEM # 8.17

The International Coffee Association has reported the mean daily coffee consumption for U.S. residents as 1.65 cups. Assume that a sample of 38 people from a North Carolina city consumed a mean of 1.84 cups of coffee per day, with a standard deviation of 0.85 cups. In a two-tail test at the 0.05 level, could the residents of this city be said to be significantly different from their counterparts across the nation? *Source: coffeeresearch.org, August 8, 2006.*

Step 5- Define your decision rule

Reject H_0 if the calculated $t < -2.026$ or > 2.026, otherwise do not reject.

Step 6- Make your decision

Calculated test statistic falls in non rejection region, do not reject H_0

PROBLEM # 8.17

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Step 7- Interpret the conclusion in context

At the 0.05 level, the mean daily coffee consumption for the residents of this North Carolina city does not differ significantly from their counterparts across the nation. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

P-Value approach

p-value = ?

TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
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At the 0.05 level, the mean daily coffee consumption for the residents of this North Carolina city does not differ significantly from their counterparts across the nation. The difference between the hypothesized population mean and the sample mean is judged to have been merely the result of chance variation.

P-Value approach

p-value/2 is between 0.05 and 0.10 which is larger than $\alpha/2 = 0.025$. Therefore p-value is larger than $\alpha = 0.05$.

Do not Reject Ho.

POPULATION PROPORTION

Bowerman, et al. (2017) pp. 406

PROBLEM # 8.19

For a simple random sample, n=200 and p=0.34. At the 0.01 level, test H_0: p_0 = 0.40 versus H_1: $p_0 \neq 0.40$.

Step 1- Set up Hypotheses

H_0: p_0 = 0.40 versus H_1: $p_0 \neq 0.40$.

Level of significance: $\alpha = 0.01$

Step 2- What is the appropriate test statistic to use?

Step 3-Calculate the test statistics value

$$z = \frac{p - p_0}{\sigma_p} = \frac{0.34 - 0.40}{\sqrt{0.4(1-0.4)/200}} = -1.73$$

Step 4- Find the critical value for the test statistic.

z = -2.58 and z = 2.58

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PROBLEM # 8.19

For a simple random sample, n=200 and p=0.34. At the 0.01 level, test H_0: $p_0 = 0.40$ versus H_1: $p_0 \neq 0.40$.

Step 5- Define your decision rule

Reject H_0 if the calculated z < -2.58 or > 2.58, otherwise do not reject.

Step 6- Make your decision

Calculated test statistic does not fall in rejection region, do not reject H_0.

Step 7- Interpret the conclusion in context

At the 0.01 level, the results suggest that the population proportion could be 0.40.

The difference between the hypothesized population proportion and the sample proportion is judged to have been merely the result of chance variation.

PROBLEM # 8.23

For each of the following situations, determine whether a one-tail test or a two-tail test would be appropriate. Describe the test, including the null and alternative hypotheses, then explain your reasoning in selecting it.

a. A machine that has not been serviced for several months is producing output in which 5% of the items are defective. The machine has just been serviced and quality should now be improved.

H_0: $p_0 \geq 0.05$ and H_1: $p_0 < 0.05$

PROBLEM # 8.23

For each of the following situations, determine whether a one-tail test or a two-tail test would be appropriate. Describe the test, including the null and alternative hypotheses, then explain your reasoning in selecting it.

b. In a speech during her campaign for reelection, a Republican candidate claims that 55% of registered Democrats in her county intend to vote for her.

H_0: $p_0 = 0.55$ and H_1: $p_0 \neq 0.55$

PROBLEM # 8.23

For each of the following situations, determine whether a one-tail test or a two-tail test would be appropriate. Describe the test, including the null and alternative hypotheses, then explain your reasoning in selecting it.

c. Of those who have bought a new car in the part, a dealer has found that 70% experience three or more mechanical problems in the first four months of ownership. Unhappy with this percentage, the dealer has heavily revised the procedure by which pre-delivery mechanical checks are carried out.

H_0: $p_0 \geq 0.70$ and H_1: $p_0 < 0.70$

PROBLEM # 8.24

What is a p-value, and how it is relevant to hypothesis testing?

A p-value is the exact level of significance associated with the calculated value of the test statistic. It is the most extreme critical value that the test statistic would be capable of exceeding. If p-value < α, reject H_0 and if p-value $\geq \alpha$, do not reject H_0.

PROBLEM # 8.25

The p-value for a hypothesis test has been reported as 0.03. If the test result is interpreted using the α=0.05 level of significance as a criterion, will H_0 be rejected? Explain.

Since p-value = 0.03 is less than α = 0.05, the null hypothesis would be rejected. The sample result is more extreme than you would have been willing to attribute to chance.

PROBLEM # 8.26

A hypothesis test is carried out using the α=0.01 level of significance and H_0 cannot be rejected. What is the most accurate statement we can make about the p-value for this test?

If we are unable to reject H_0, then the p-value is not less than the level of significance being used (α = 0.01), or p-value $\geq$ 0.01.