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LESSON 5
CONTINUOUS PROBABILITIES

SAMIE L.S. LY

LESSON 4/5 IN A NUTSHELL

DISCRETE PROBABILITIES

FREQUENCY DISTRIBUTION CALCULATIONS

BINOMIAL PROBABILITY FUNCTION

- UNDERSTAND CONTEXTS

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

CONTINUOUS PROBABILITES

NORMAL PROBABILITY DISTRIBUTION

- UNDERSTAND Z-TABLE

- **Normal Probability Distribution**
- **Standard Normal Probability Distribution**
- **Computing Probabilities**

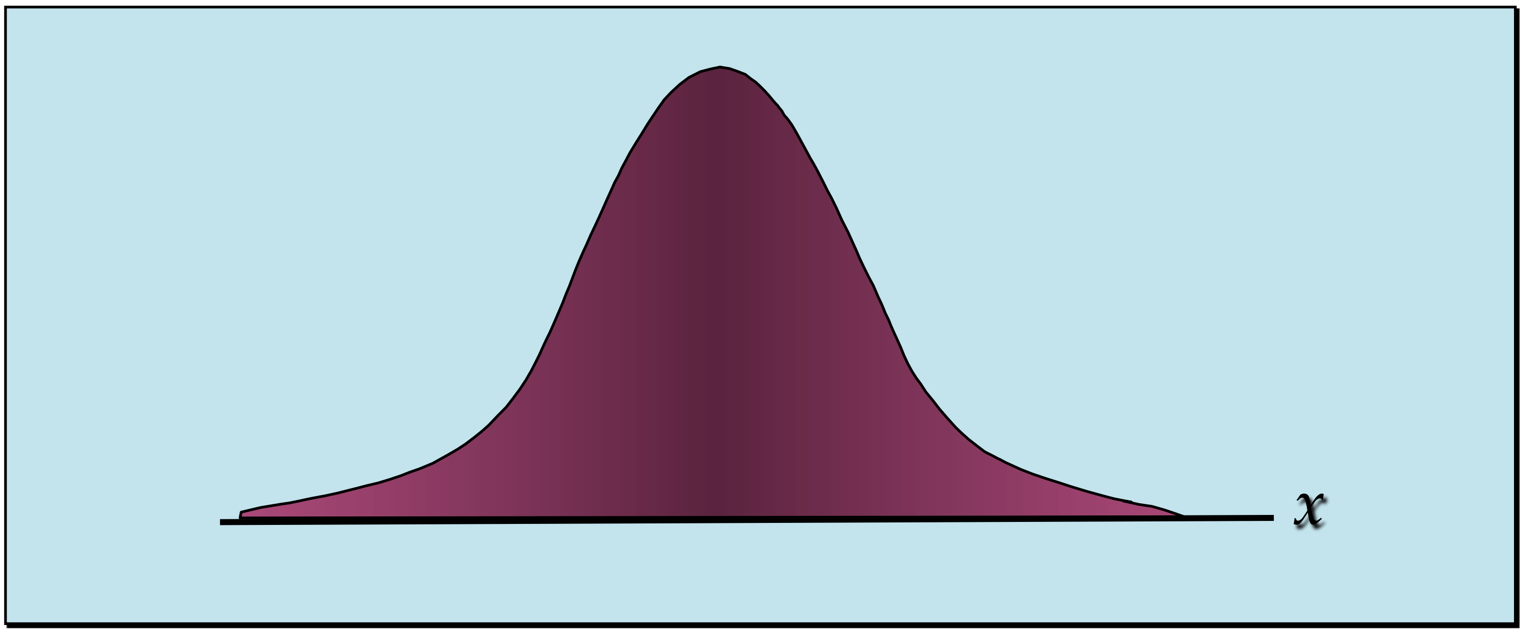
NORMAL CURVE

Some Characteristics

Bowerman, et al. (2017) pp. 289

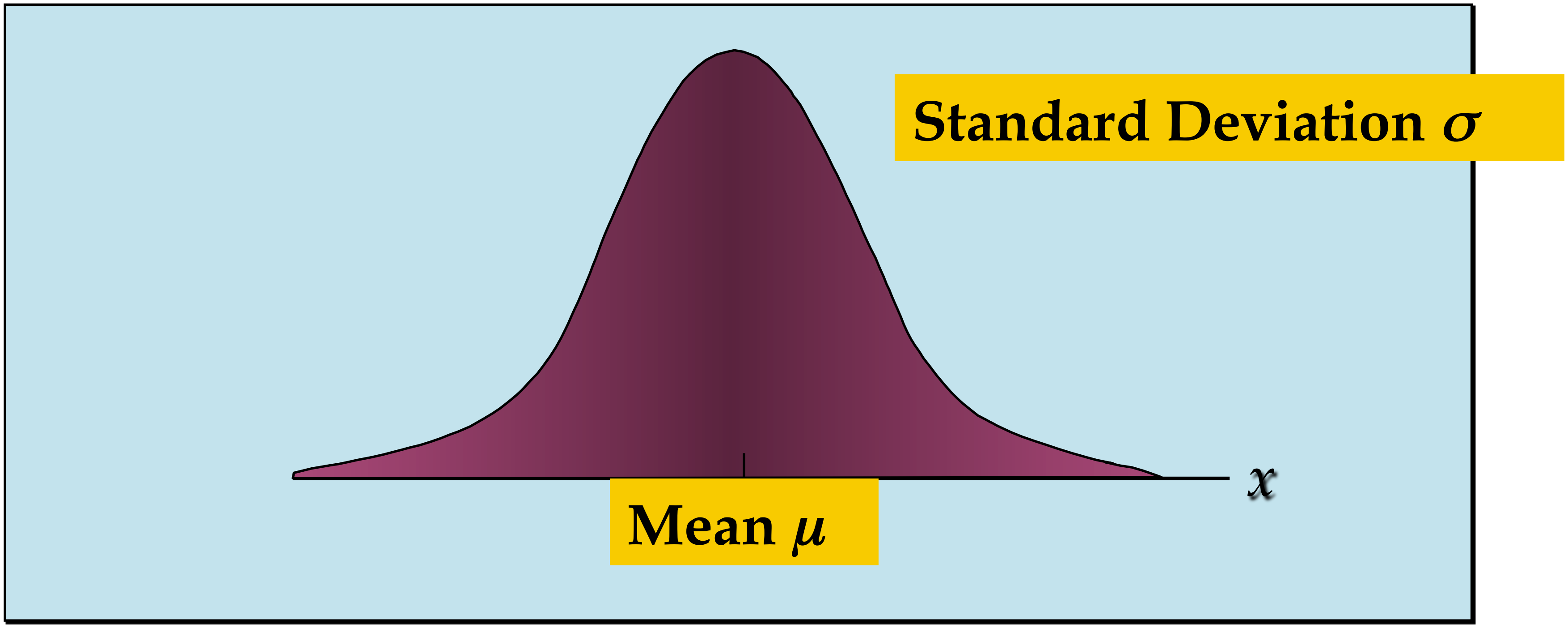
NORMAL CURVE

Symmetric Distribution

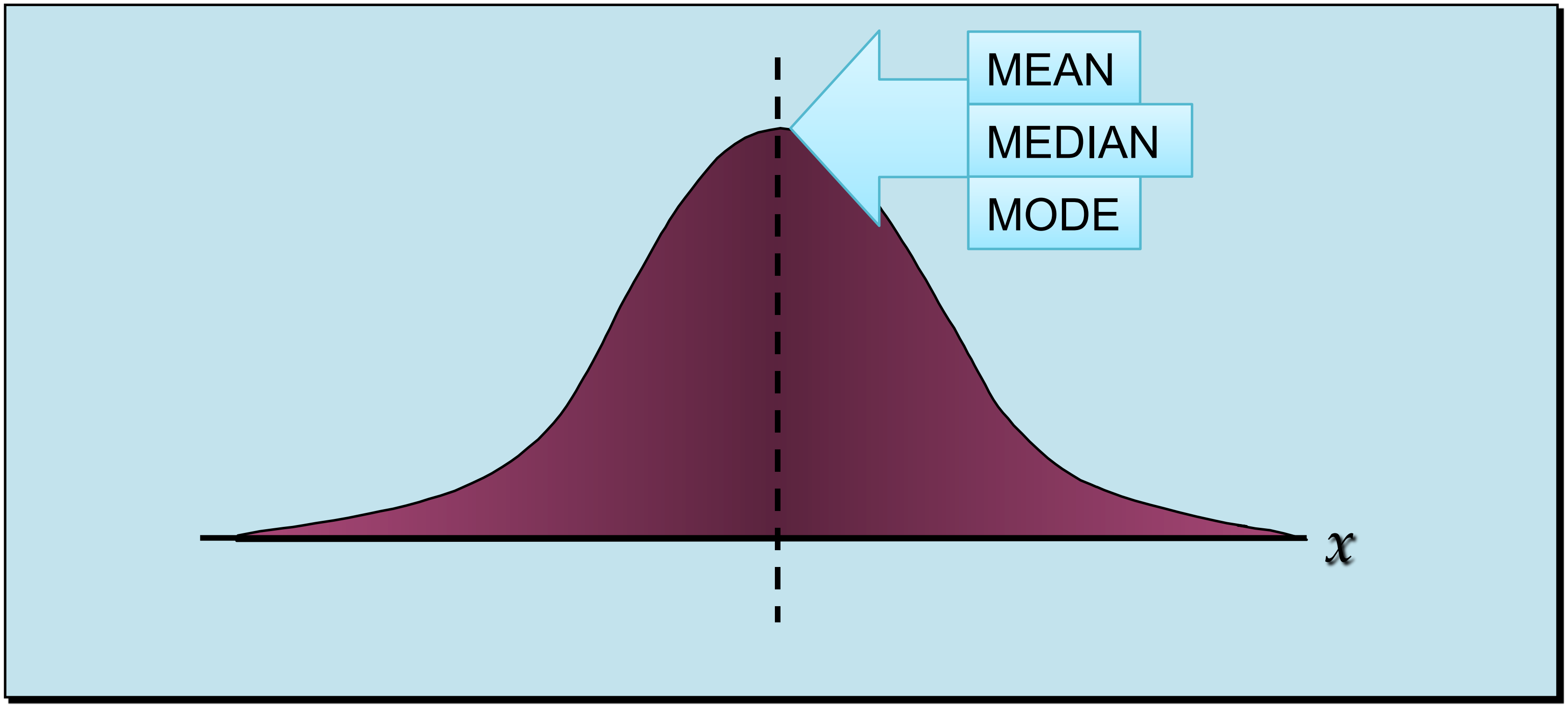


Skewness measure = 0

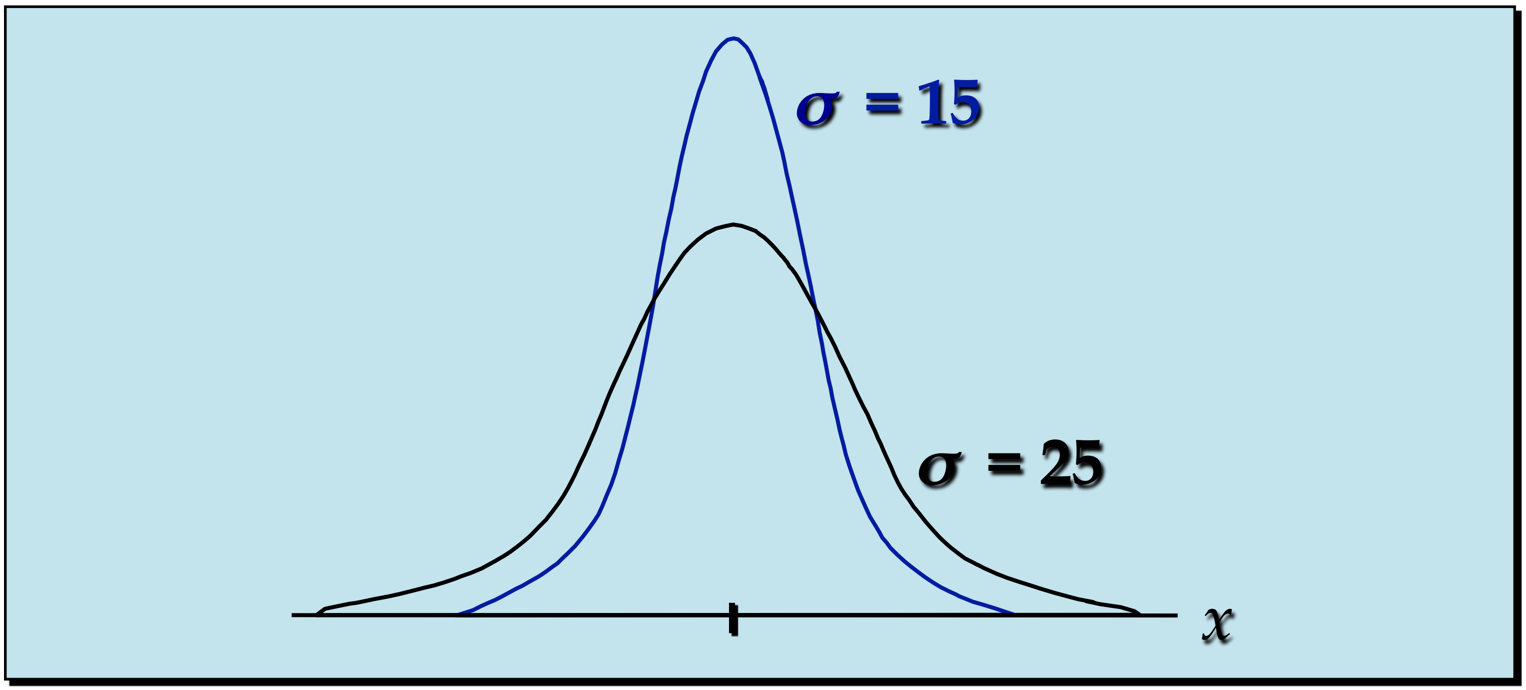
NORMAL CURVE



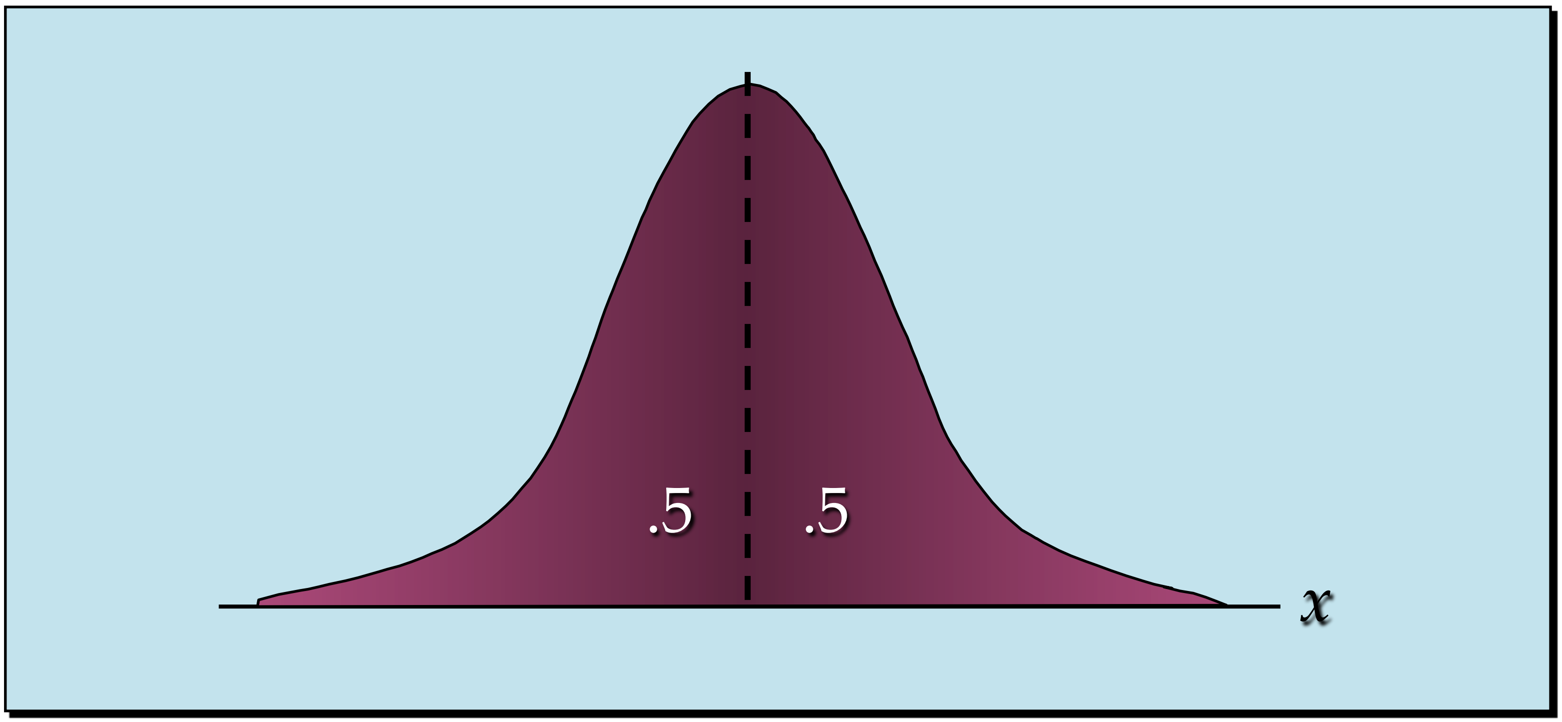
NORMAL CURVE



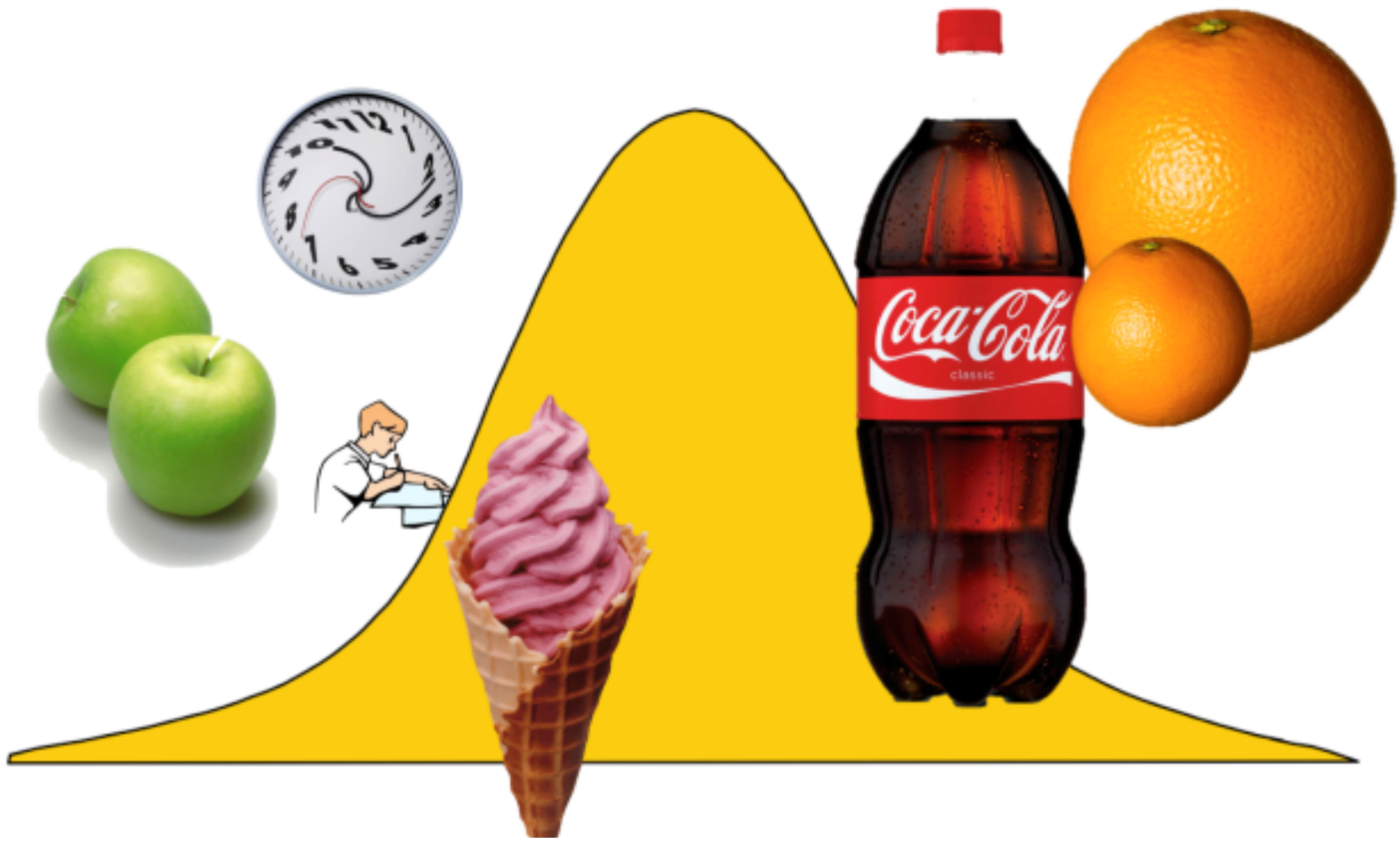
NORMAL CURVE



NORMAL CURVE



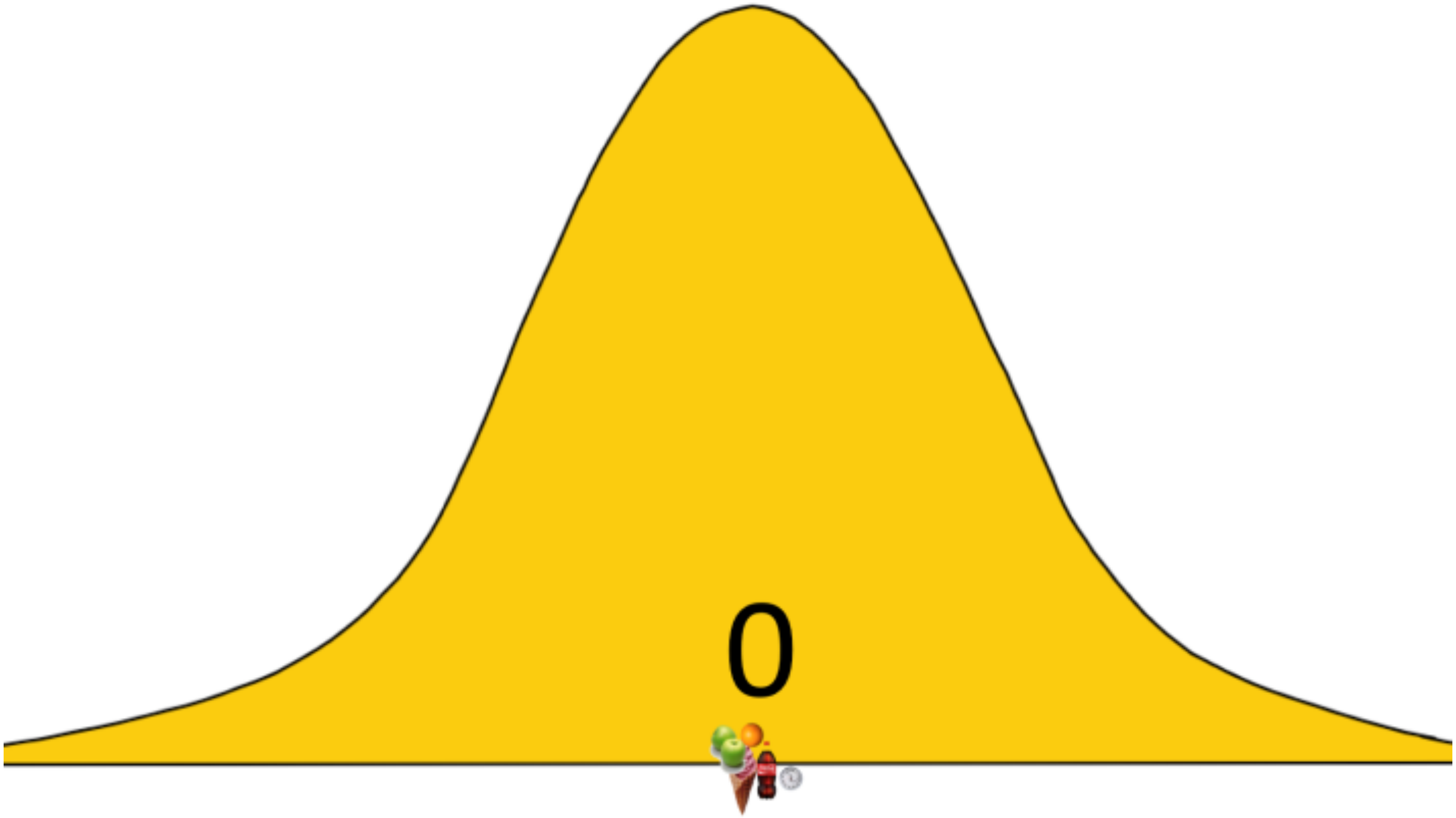
STANDARD NORMAL PROBABILITY DISTRIBUTION



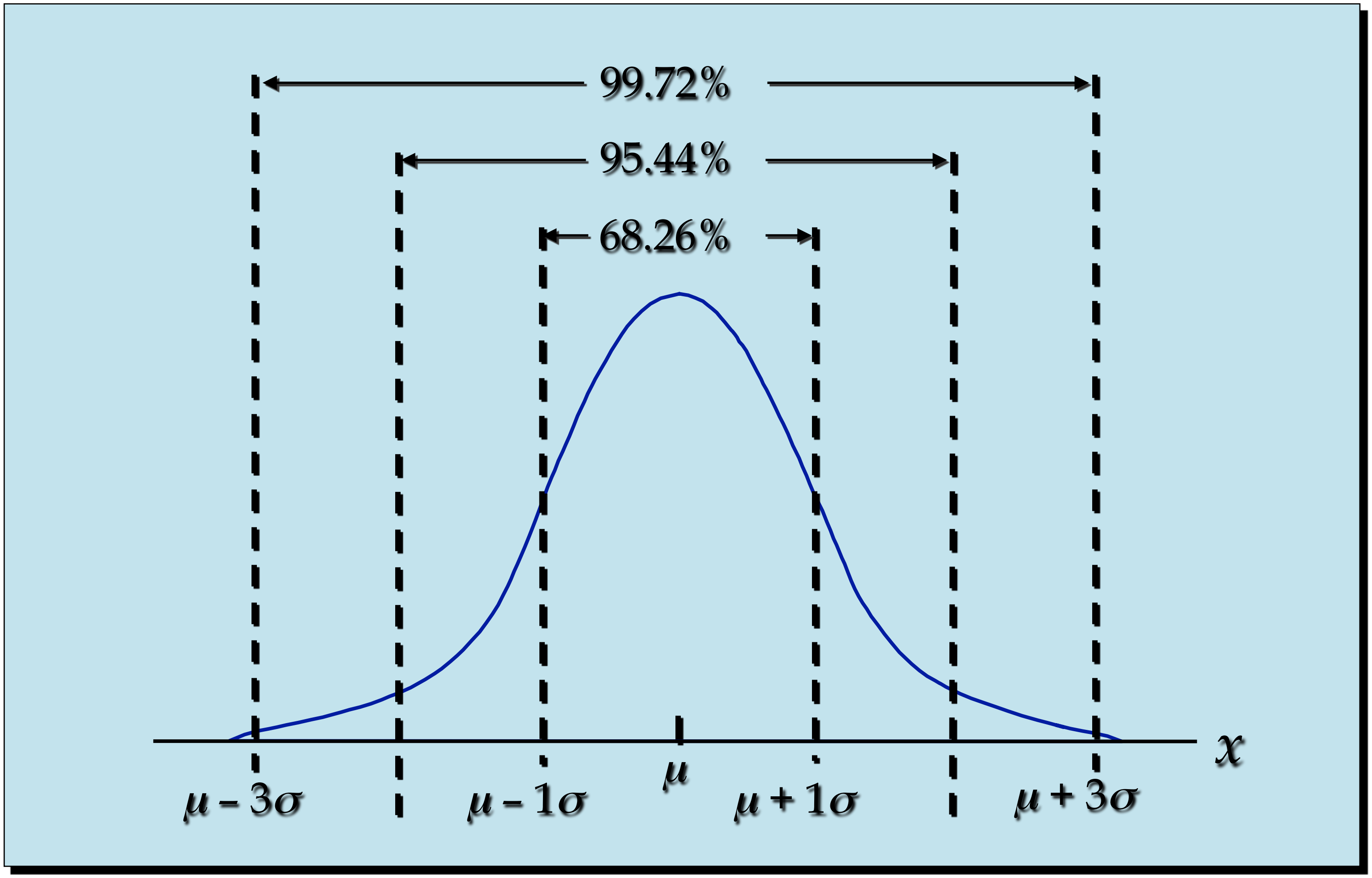
Standard Normal Probability Distribution

Computing Probabilities

STANDARD NORMAL PROBABILITY DISTRIBUTION



AREAS UNDER THE CURVE



Bowerman, et al. (2017) pp. 295

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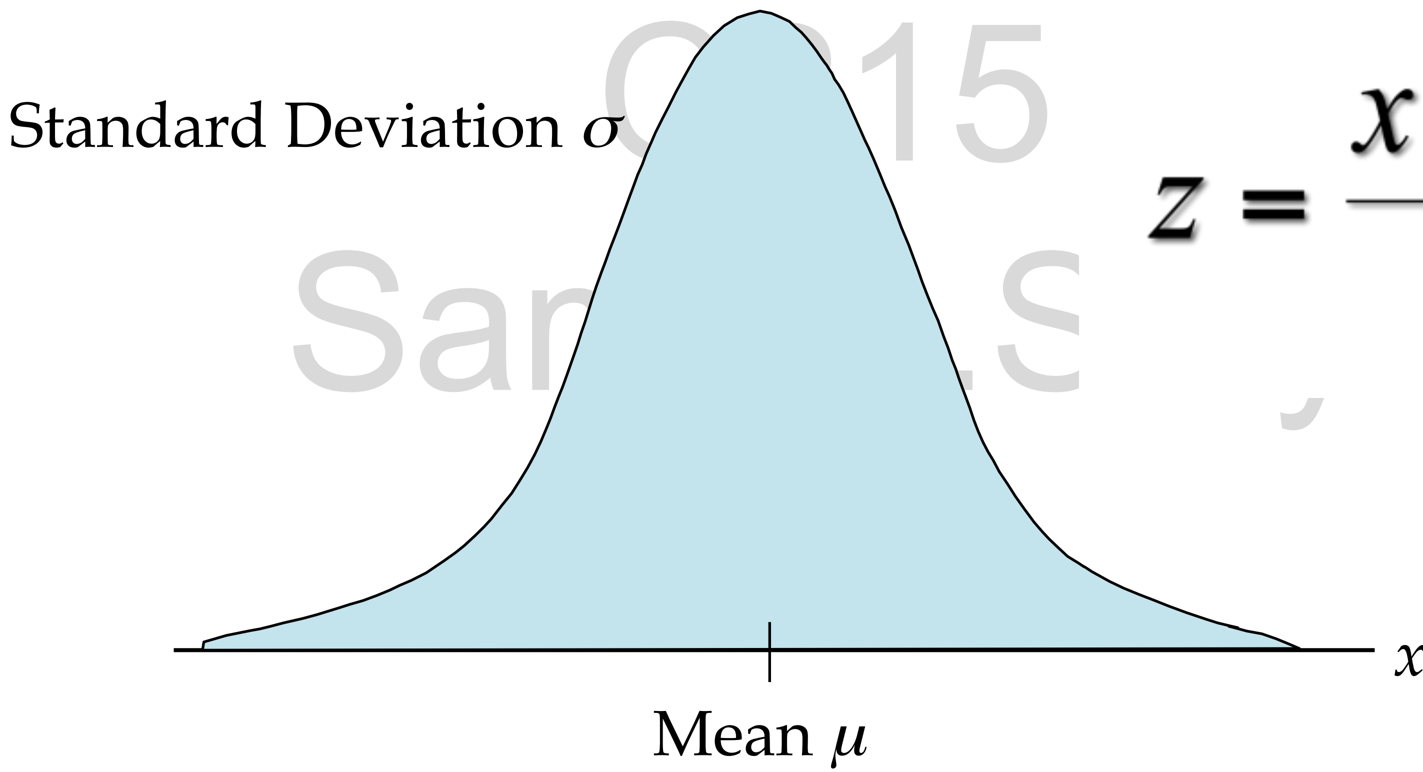
STANDARD NORMAL PROBABILITY DISTRIBUTION

Think of z as a measure of the number of standard deviations x is from μ

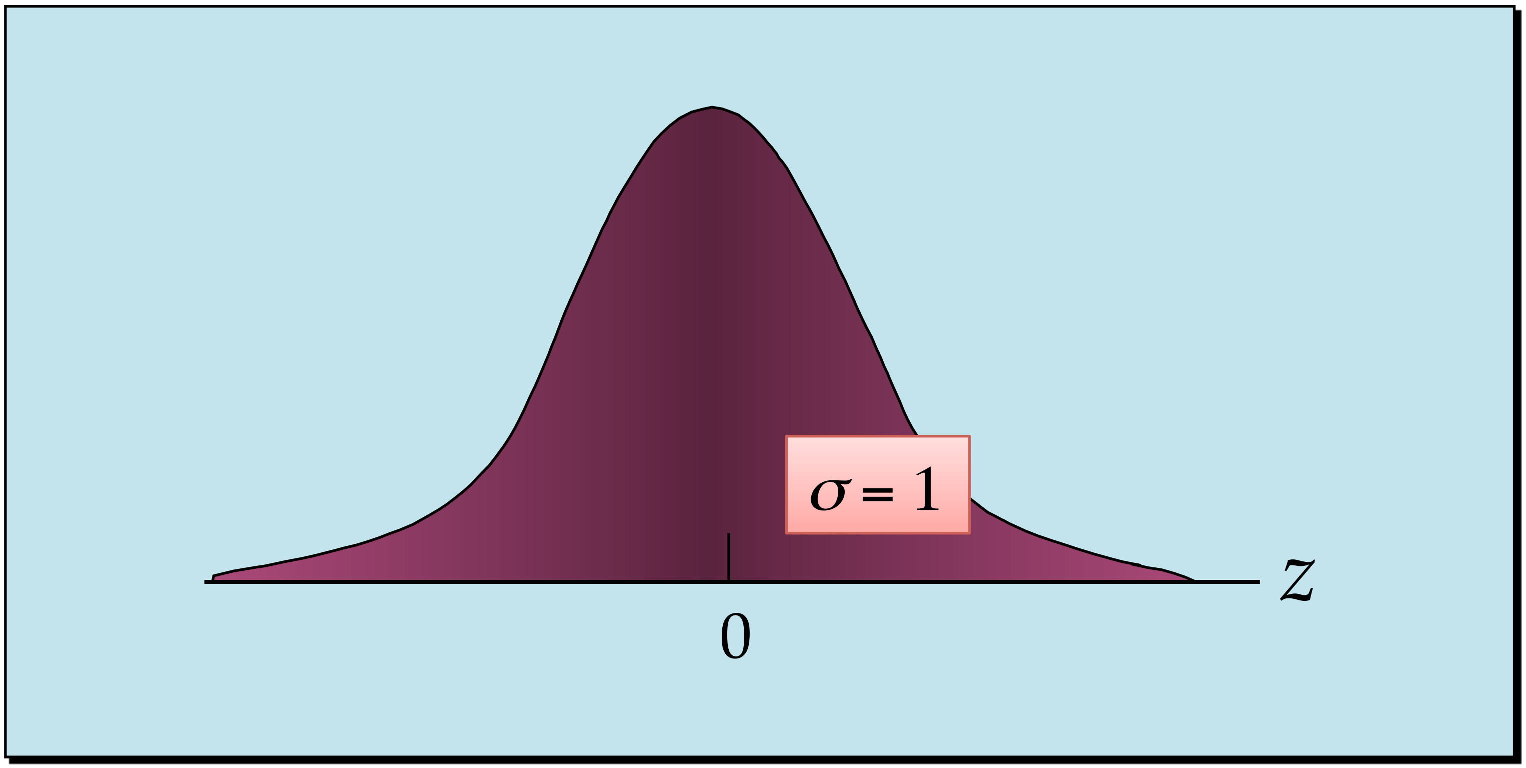


$$z = \frac{x - \mu}{\sigma}$$

Bowerman, et al. (2017) pp. 296

STANDARD NORMAL PROBABILITY DISTRIBUTION

Letter z is used to designate the standard normal random variable



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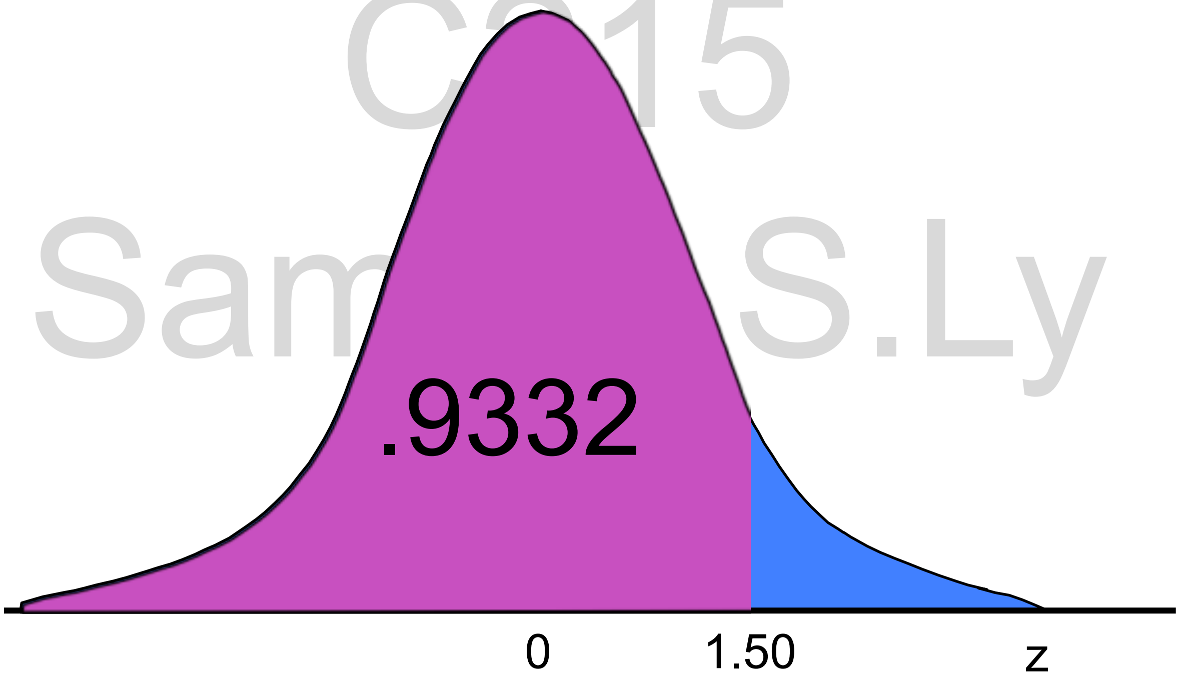
Computing Probabilities

PROBLEM # 5.1

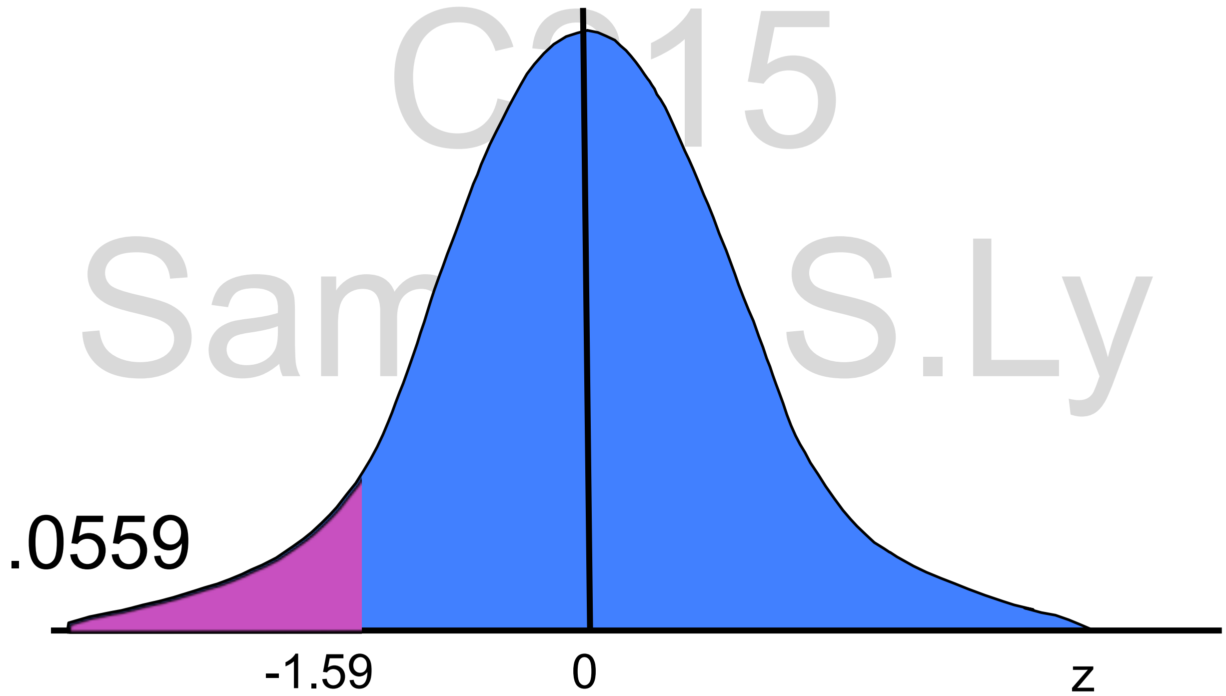
$P(Z < 1.50) =$

.9332



PROBLEM # 5.2

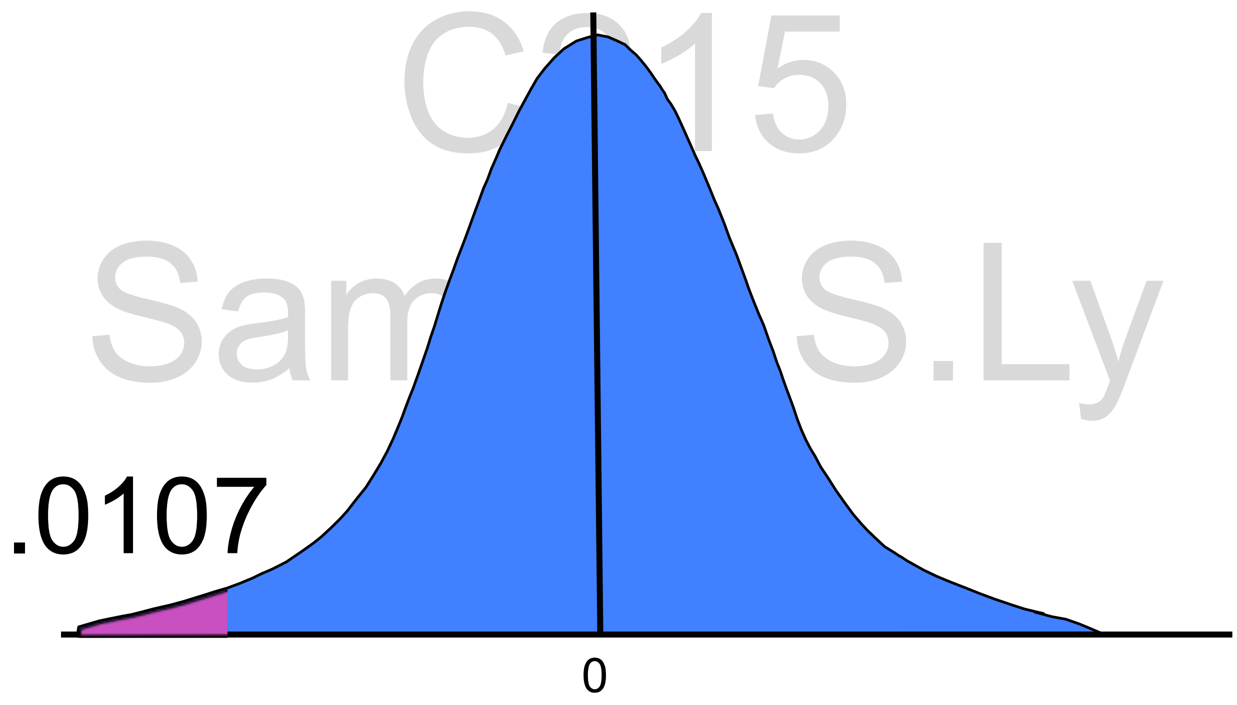
P (Z < -1.59) =



PROBLEM # 5.3

P (Z < -2.30) =

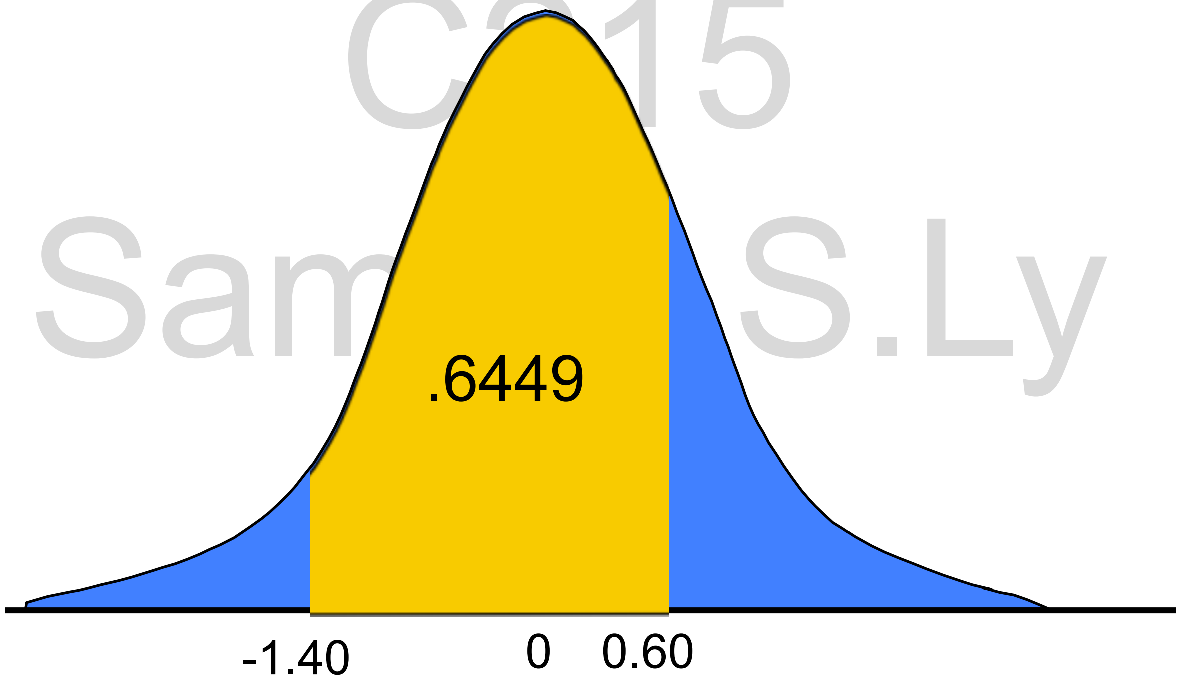
.0107



PROBLEM # 5.4

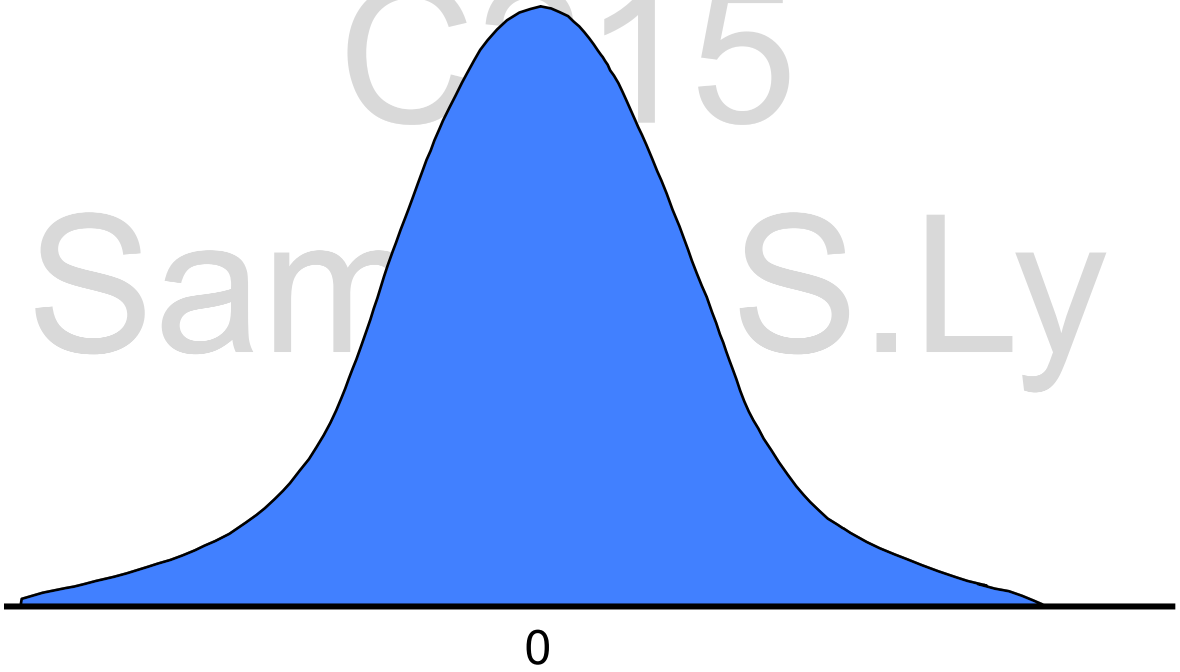
P (-1.40 < Z < .60)=

.6449



PROBLEM # 5.7

P (Z > 4.0) = 0

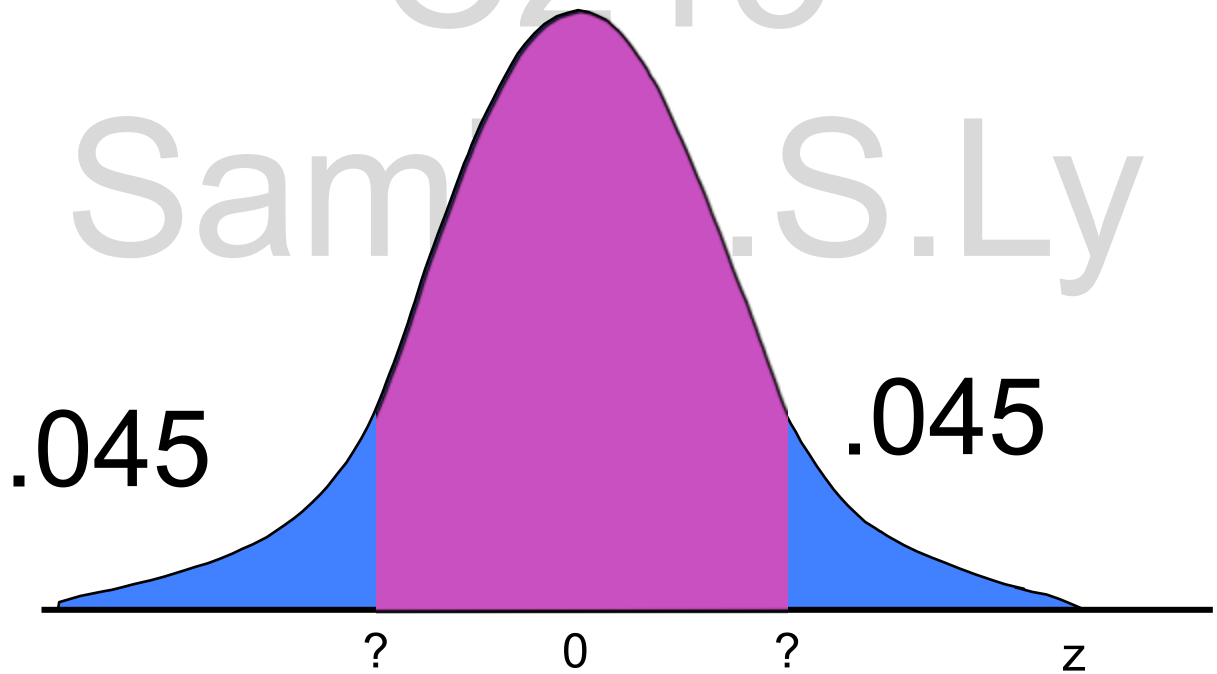


PROBLEM # 5.10

Find $z_{.045}$

$P(Z < z_{.045})$ or $P(Z > z_{.045})$

$P(z=-1.70)$, $P(z=1.70)$

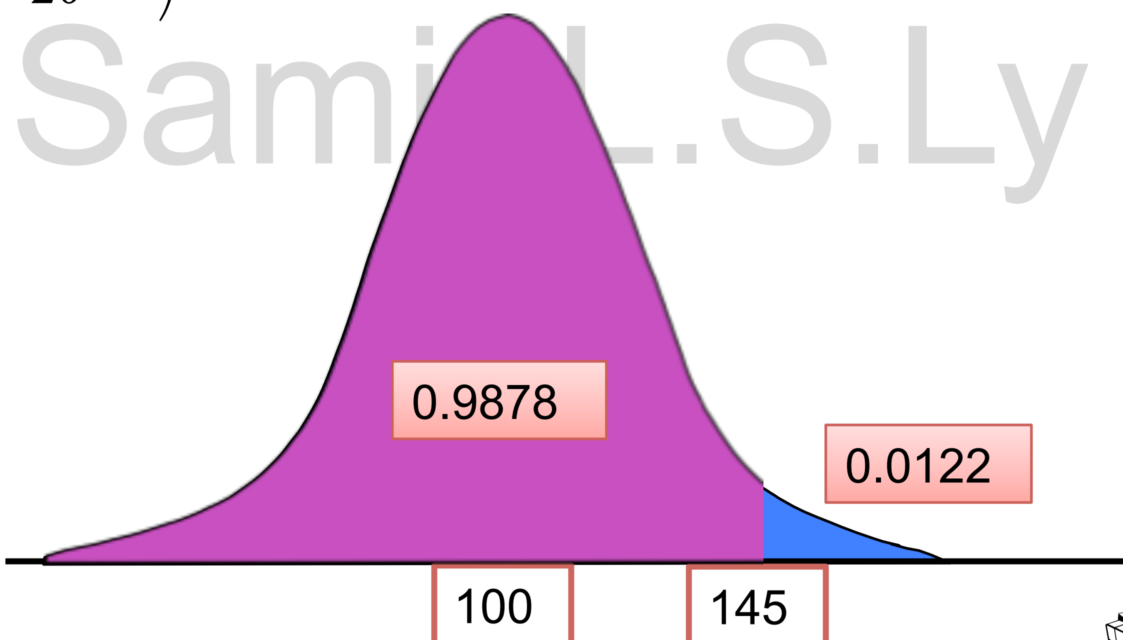


PROBLEM # 5.11

X is normally distributed with a mean of 100 and a standard deviation of 20. What is the probability that X is greater than 145?

P(X > 145)

$$= P\left(\frac{X-\mu}{\sigma} > \frac{145-100}{20}\right) = P(Z > 2.25) = 1 - P(Z < 2.25) = 1 - .9878 = .0122$$



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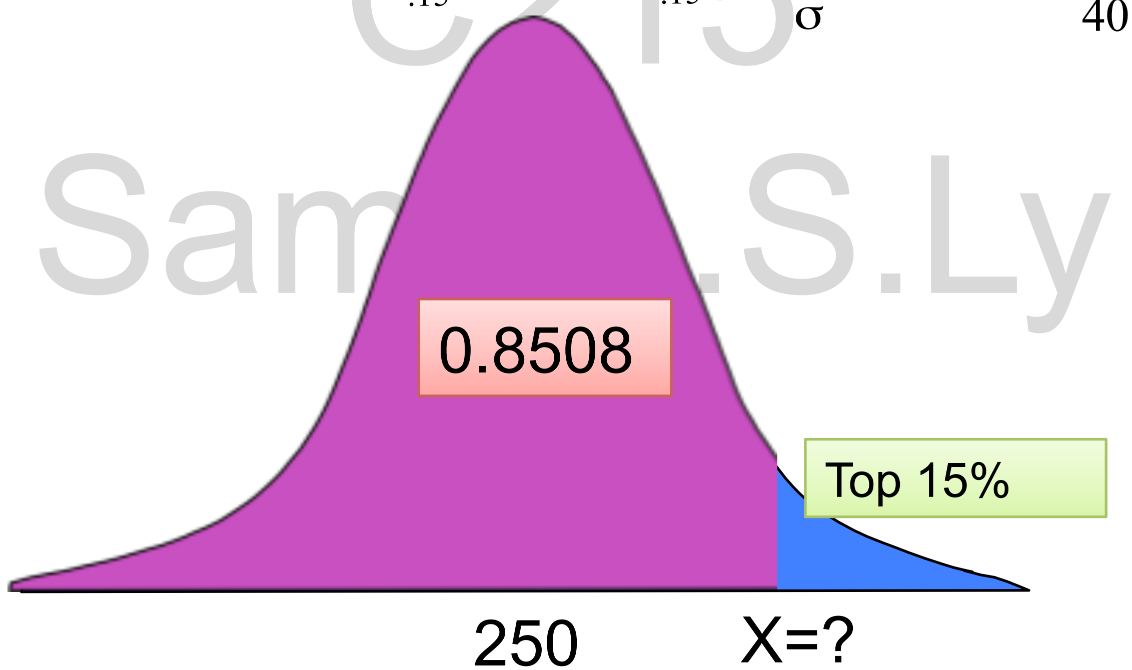
PROBLEM # 5.13

X is normally distributed with a mean of 250 and a standard deviation of 40. What value of X does only the top 15% exceed?

$P(Z < z_{.15}) = 1 - .15 = .8500$; $z_{.15} = 1.04$; $z_{.15} = \dfrac{x-\mu}{\sigma}$; $1.04 = \dfrac{x-250}{40}$; $x = 291.6$



PROBLEM # 5.15

The long-distance calls made by the employees of a company are normally distributed with a mean of 6.3 minutes and a standard deviation of 2.2 minutes. Find the probability that a call

a. Lasts between 5 and 10 minutes

$$P(5 < X < 10) = P\left(\frac{5-6.3}{2.2} < \frac{X-\mu}{\sigma} < \frac{10-6.3}{2.2}\right) = P(-.59 < Z > 1.68)$$

$$= P(Z < 1.68) - P(Z < -.59) = .9535 - .2776 = .6759$$

b. Lasts less than 4 minutes

$$P(X < 4) = P\left(\frac{X-\mu}{\sigma} < \frac{4-6.3}{2.2}\right) = P(Z < -1.05) = .1469$$

PROBLEM # 5.15

The long-distance calls made by the employees of a company are normally distributed with a mean of 6.3 minutes and a standard deviation of 2.2 minutes. Find the probability that a call

d. How long do the longest 10% of calls last?

$$P(Z < z_{.10}) = 1 - .10 = .9000;$$

$$z_{.10} = 1.28; \; z_{.10} = \frac{x - \mu}{\sigma}; \; 1.28 = \frac{x - 6.3}{2.2}; \; x = 9.116$$

Calls last at least 9.116 minutes.

PROBLEM # 5.16

The lifetime of light bulbs that are advertised to last for 5,000 hours are normally distributed with a mean of 5,100 hours and a standard deviation of 200 hours. What is the probability that a bulb lasts longer than the advertised figure?

$$P(X > 5{,}000) = P\left(\frac{X-\mu}{\sigma} > \frac{5{,}000 - 5{,}100}{200}\right) = P(Z > -.5)$$

$$= 1 - P(Z < -.5) = 1 - .3085 = .6915$$

PROBLEM # 5.18

How much money does a typical family of four spend at McDonald's restaurants per visit? The amount is a normally distributed random variable whose mean is $16.40 and whose standard deviation is $2.75.

a. Find the probability that a family of four spends less than $10.

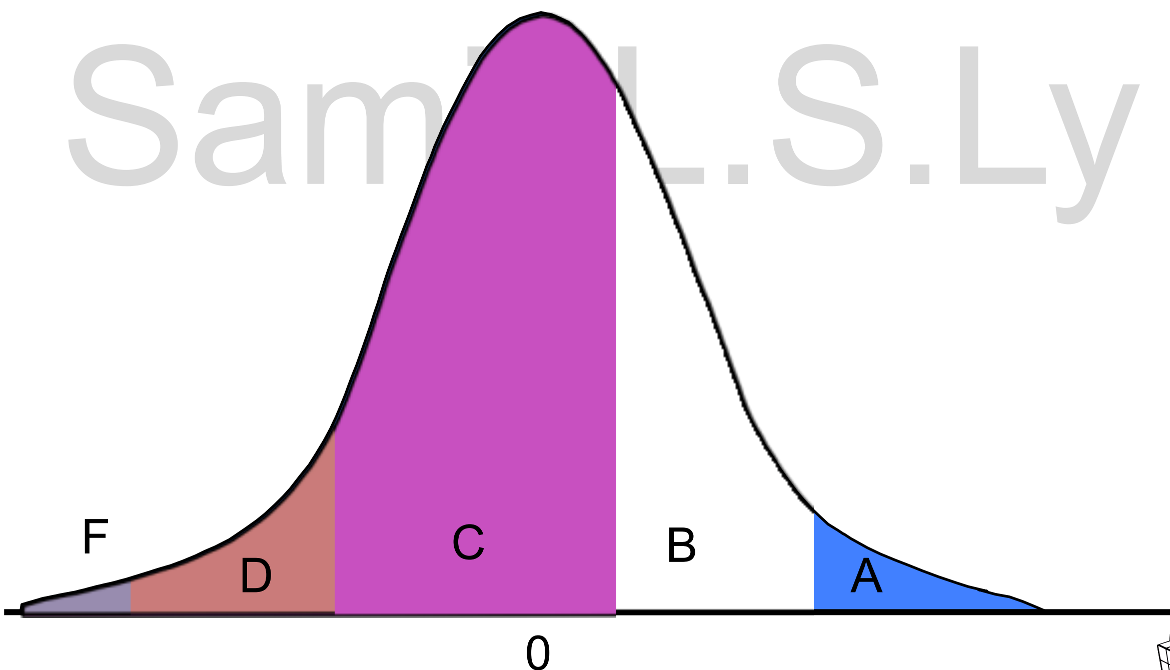
$$P(X < 10) = P\left(\frac{X-\mu}{\sigma} < \frac{10-16.40}{2.75}\right) = P(Z < -2.33) = .0099$$

b. What is the amount below which only 10% of families of four spend at McDonald's?

$$P(Z < -z_{.10}) = .1000;\ -z_{.10} = -1.28;\ -z_{.10} = \frac{x-\mu}{\sigma};\ -1.28 = \frac{x-16.40}{2.75};\ x = 12.88$$

PROBLEM # 5.19

The final marks in a statistics course are normally distributed with a mean of 70 and a standard deviation of 10. The professor must convert all marks to letter grades. She decides that she wants 10% A's, 30% B's, 40% C's, 15% D's, and 5% F's. Determine the cut offs for each letter grade.



PROBLEM # 5.19

The final marks in a statistics course are normally distributed with a mean of 70 and a standard deviation of 10. The professor must convert all marks to letter grades. She decides that she wants 10% A's, 30% B's, 40% C's, 15% D's, and 5% F's. Determine the cut offs for each letter grade.

A: $P(Z < z_{.10}) = 1 - .10 = .9000;\ z_{.10} = 1.28;\ z_{.10} = \frac{x-\mu}{\sigma};\ 1.28 = \frac{x-70}{10};\ x = 82.8$

B: $P(Z < z_{.40}) = 1 - .40 = .6000;\ z_{.40} = .25;\ z_{.40} = \frac{x-\mu}{\sigma};\ .25 = \frac{x-70}{10};\ x = 72.5$

C: $P(Z < -z_{.20}) = .2000;\ -z_{.20} = -.84;\ -z_{.20} = \frac{x-\mu}{\sigma};\ -.84 = \frac{x-70}{10};\ x = 61.6;$

D: $P(Z < -z_{.05}) = .0500;\ -z_{.05} = -1.645;\ -z_{.05} = \frac{x-\mu}{\sigma};\ -1.645 = \frac{x-70}{10};\ x = 53.55$

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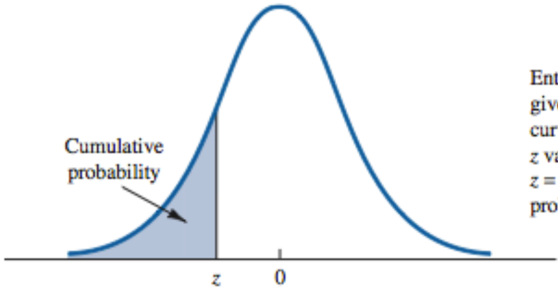
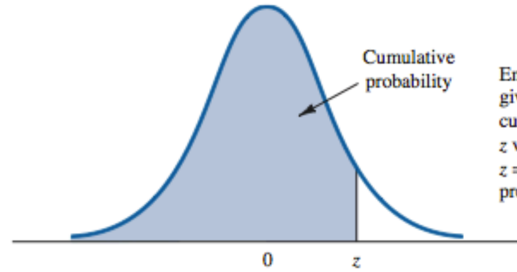
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990