COMM215

First the Foundation, then Innovation

LESSON 3
PROBABILITIES

SAMIE L.S. LY

PRACTICE 1

If the probability of A is 0.45 and the probability of the intersection of A and B is 0.15, then the probability that B will occur given that A has occurred is?

PRACTICE 2

The probability of stock A rising is 0.3 and of stock B is 0.4. If stocks A and B are not independent and the probability of both stocks rising is 0.09, what is the probability that neither stock rises?

PRACTICE PROBLEMS FOR MIDTERM

A commission studying employment in towns hard hit by a recession has collected data on which the

following questions have been based. In Town A, twenty percent of the work force is unemployed. Sixty percent of the unemployed are women, and thirty percent of the women in the work force are unemployed.

a. What percentage of the workforce is female?

b. What percentage of the men in the workforce is unemployed?

- **Experiments, Counting Rules, and Assigning Probabilities**
- **Events and their probability**
- **Some Basic Probability Laws**
- **Conditional Probability**

PROBABILITY

What are the chances that sales will decrease if we increase prices?

What is the likelihood a new assembly method will increase productivity?

How likely is it that the project will finish on time?

What is the chance that a new investment will be profitable?

Bowerman, et al. (2017) pp. 221

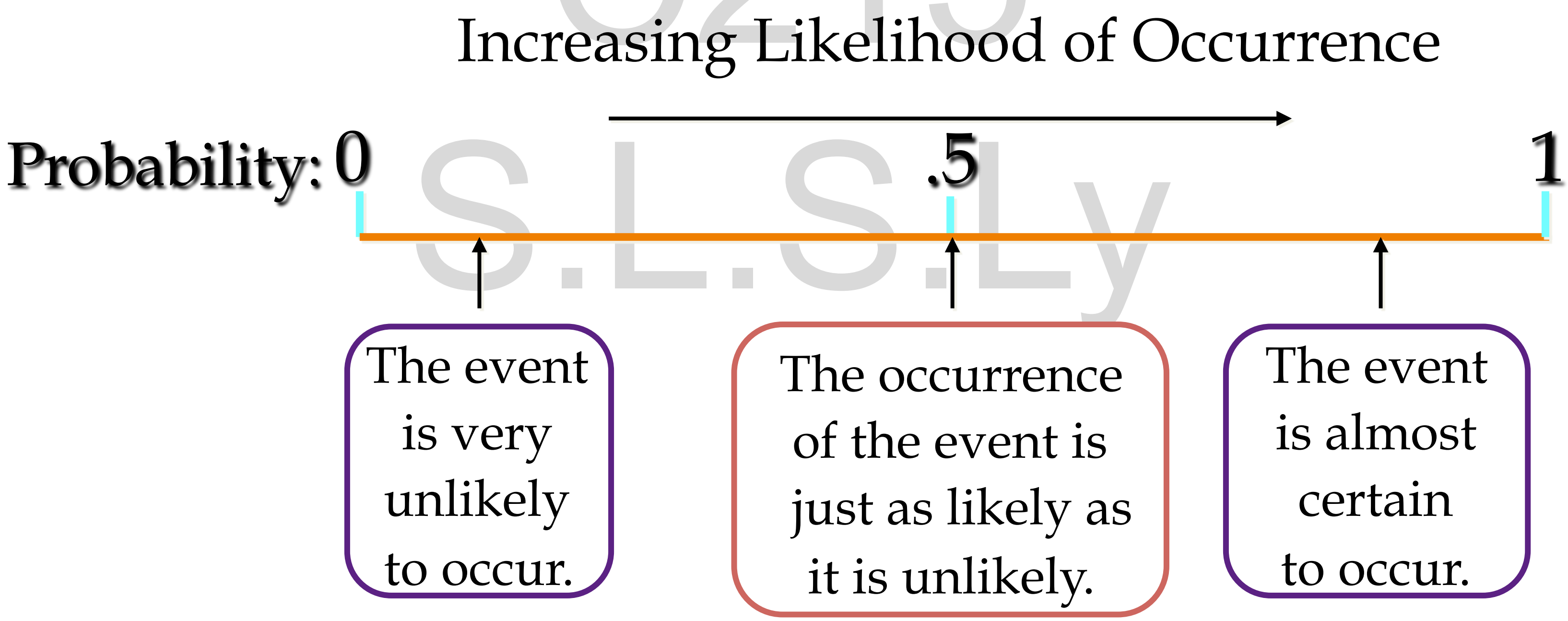
PROBABILITY

Probability is a numerical measure of the likelihood that an event will occur.

Probabilities can be used as measures of uncertainty.

PROBABILITY VALUES

are always assigned on a scale from 0 to 1



Bowerman, et al. (2017) pp. 221

EXPERIMENTS, COUNTING RULES

EXPERIMENTS

SAMPLE SPACE

SAMPLE POINT

COUNTING RULES, COMBINATIONS AND PERMUTATIONS

MULTIPLE STEP EXPERIMENT

TREE DIAGRAM

COMBINATIONS, PERMUTATIONS

EXPERIMENTS, COUNTING RULES

ASSIGNING PROBABILITIES

BASIC REQUIREMENTS

3 METHODS

- CLASSICAL
- RELATIVE FREQUENCY
- SUBJECTIVE

Bowerman, et al. (2017) pp. 222

SAMPLE SPACE

THE SET OF ALL EXPERIMENTAL OUTCOMES.

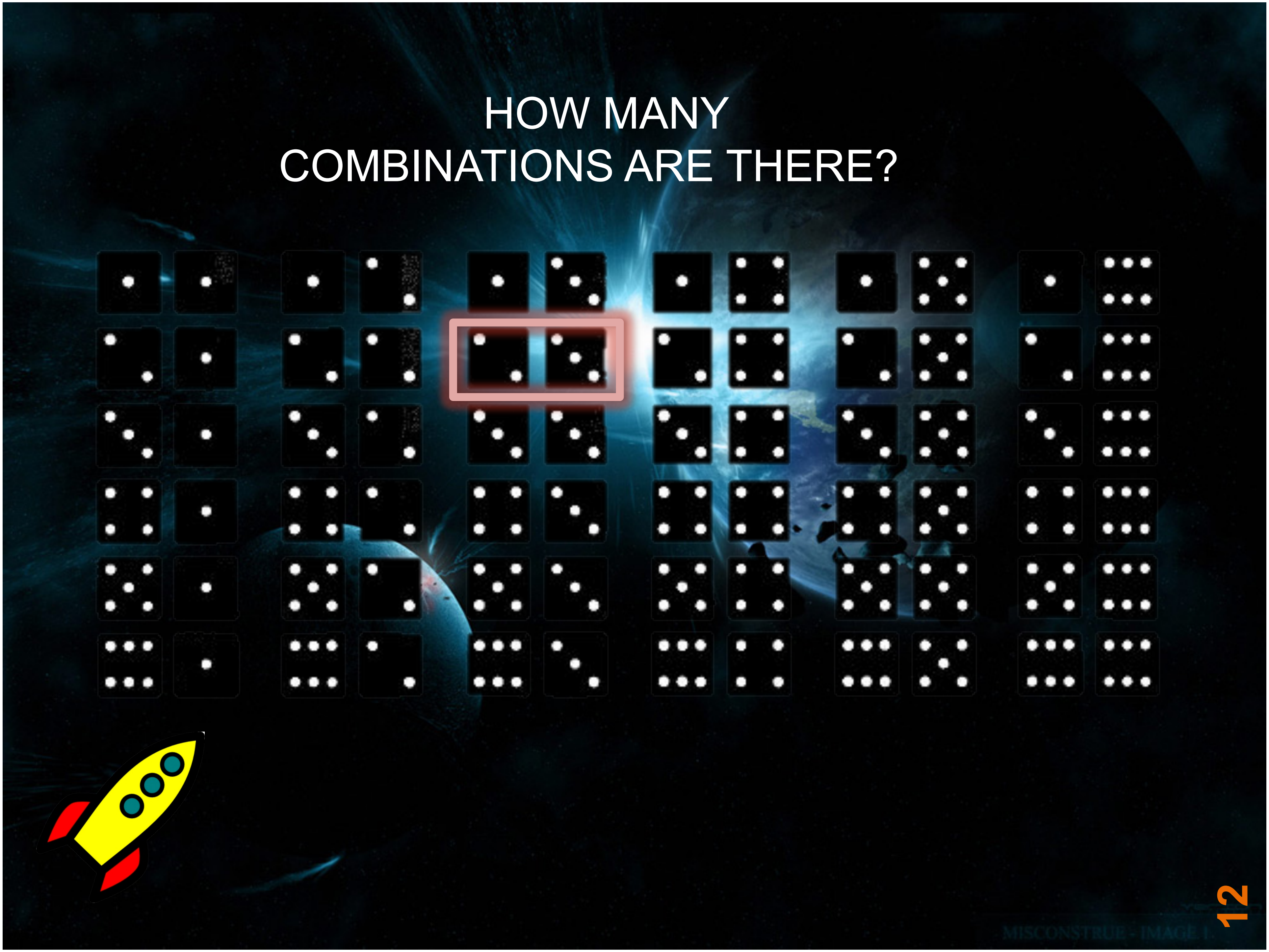
WHAT ARE ALL THE POSSIBLE OUTCOMES OF A DIE?

$$S = \{ 1, 2, 3, 4, 5, 6 \}$$

SAMPLE POINT

WHAT ABOUT TWO DICE?

Bowerman, et al. (2017) pp. 222


HOW MANY
COMBINATIONS ARE THERE?
12

PROBLEM # 3.1

Four candidates are running for mayor. The four candidates are Adams, Brown, Collins, and Dalton. Determine the sample space of the results of the election.

S= {Adams wins. Brown wins, Collins wins, Dalton wins}

MULTIPLE-STEP EXPERIMENTS

COUNTING RULE FOR MULTIPLE-STEP EXPERIMENTS:

S = { 1, 2, 3, 4, 5, 6 }

FOLLOWING *k* STEPS

Step 1- 6 possibilities

Step 2- 6 possibilities

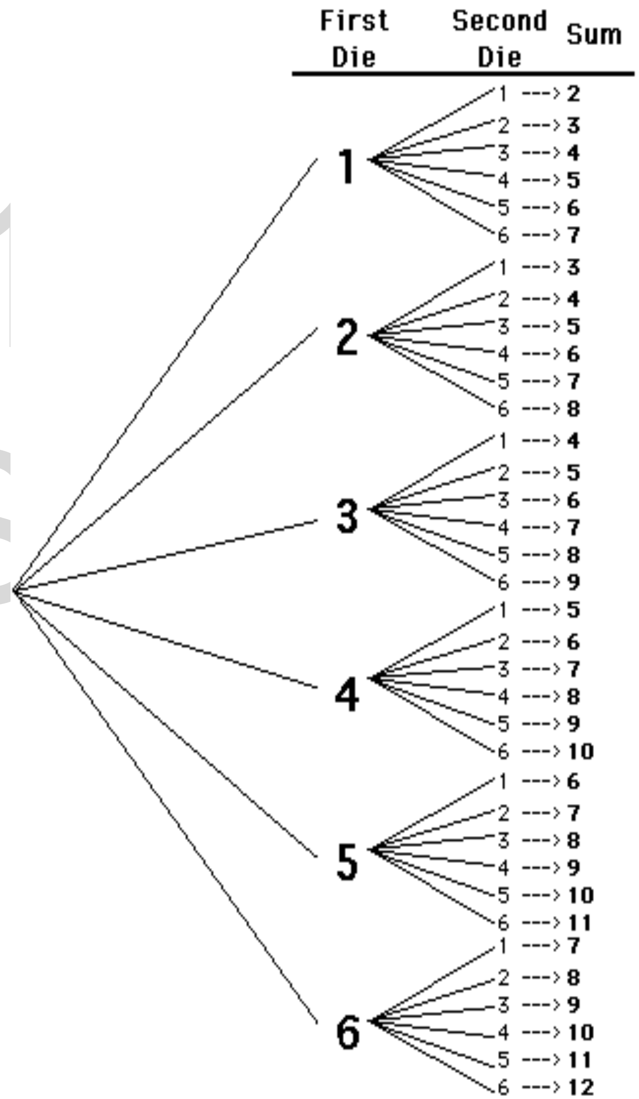
(6)(6) = 36

k = 2, (6) is repeated 2 times

Bowerman, et al. (2017) pp. 224

TREE DIAGRAM

Tree Diagram is a visual display of a multiple-step experiment.



Bowerman, et al. (2017) pp. 225

COMBINATION RULE

Combination rule allows us to count the number of experimental outcomes when the experiment involves selecting n objects from a set N.

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Order does **NOT** Matter

AB = BA

Bowerman, et al. (2017) pp. 248

COMBINATION RULE

AB = BA

N = 5

n = 2

A B C D E

A A	B A	C A		A
A B	B B	C B		B
A C	B C	C C	D C	E C
A D	B D	C D	D D	E D
A E	B E	C E	D E	E E

There are 10 combinations

PERMUTATION RULE

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

Order **MATTERS!**

AB ≠ BA

PERMUTATION RULE

There are 20 combinations

$N = 5$

$n = 2$

A B C D E

A A	B A	C A	D A	E A
A B	B B	C B	D B	E B
A C	B C	C C	D C	E C
A D	B D	C D	D D	E D
A E	B E	C E	D E	E E

$AB \neq BA$

COMBINATION VS PERMUTATION

Which are the possible dice combinations for 2 dice?

Position 1	Position 2

Which are the possible sum values for 2 dice?

Value 1	Value 2

PROBLEM # 3.4

Investing in stocks. From a list of 15 preferred stocks recommended by your broker, you will select three to invest in. How many different ways can you select the three stocks from the 15 recommenced stocks?

The number of ways would be

$$\binom{15}{3} = \frac{15!}{3!\,(15-3)!} = \frac{15 * 14 * 13 \ldots 3 * 2 * 1}{3 * 2 * 1 * 12 * 11 * 10 \ldots 3 * 2 * 1} = \frac{1.3076 * 10^{12}}{2874009600} = 455$$

ASSIGNING PROBABILITIES

Basic Requirements:

1. **The probability assigned to each experimental outcome must be between 0 and 1.**

$$0 \leq P(E_i) \leq 1 \quad \text{for all } i$$

2. **The sum of the probabilities for all the experimental outcomes must be equal to 1.0.**

$$P(E_1) + P(E_2) + \ldots + P(E_n) = 1$$

THE CLASSICAL METHOD

If an experiment has *n* possible outcomes,

Each outcome would consist of 1/n.

For example: Rolling a Die

Sample Space: S={ 1, 2, 3, 4, 5, 6 }

Each sample point has 1/6 of occurring

THE RELATIVE FREQUENCY METHOD

GOAL IN YOUR MAJOR

GOAL	FREQUENCY	RELATIVE FREQUENCY	PERCENT FREQUENCY
COMPANY	2	0.02	2.17
GRAD	26	0.28	28.26
JOB	49	0.53	53.26
NO CLUE	11	0.12	11.96
PROMOTION	3	0.03	3.26
SWITCH	1	0.01	1.09
	92	1.00	100

THESE ARE ALL PROBABILITIES!

26/92 = 0.28

THE SUBJECTIVE METHOD

Appropriate to use:

- **WHEN YOU CANNOT MAKE A REALISTIC ASSUMPTION OF THE EXPERIMENTAL OUTCOMES**
- **WHEN YOU HAVE LITTLE RELEVANT DATA.**

Different people can be expected to assign different probabilities.

Must follow the basic requirements of assigning probabilities.

PROBLEM #3.5

An investor tells you that in her estimation there is 60% probability that the Dow Jones Industrial Index will increase tomorrow.

a. Which approach was used to produce this figure?

Subjective approach

b. Interpret the 60% probability.

The Dow Jones Industrial Index will increase on 60% of the days if economic conditions remain unchanged

PROBLEM# 3.7

A quiz contains a multiple-choice question with five possible answers, only one of which is correct. A student plans to guess the answer because he knows absolutely nothing about the subject.

a. Produce the sample space for each question

{a is correct, b is correct, c is correct, d is correct, e is correct}

b. Assign probabilities to the simple events in the sample space you produced.

P(a is correct) = P(b is correct) = P(c is correct) = P(d is correct) = P(e is correct) = .2

PROBLEM# 3.7

A quiz contains a multiple-choice question with five possible answers, only one of which is correct. A student plans to guess the answer because he knows absolutely nothing about the subject.

c. Which approach did you use to answer part (b)?

Classical approach

d. Interpret the probabilities you assigned in part (b).

In the long run all answers are equally likely to be correct.

- Events and their probability
- Some Basic Probability Laws
- Conditional Probability

EVENTS AND THEIR PROBABILITIES

An **EVENT** is a collection of sample points.

Event C= {……..}

The **PROBABILITY OF AN EVENT** is equal to the sum of the probabilities of the sample points in the event.

Probability of Event C is P(C)

The SAMPLE SPACE is an event.
P(S) = 1.

PROBLEM # 3.8

Sample Points	Probabilities
1	.05
2	.20
3	.30
4	.30
5	.15

The sample space for an experiment contains five sample points with probabilities as shown in the table. Find the probability of each of the following events:

A: {Either 1,2, or 3 occurs }

$$P(A) = P(1) + P(2) + P(3) = .05 + .20 + .30 = .55$$

B: {Either 1,3, or 5 occurs }

$$P(B) = P(1) + P(3) + P(5) = .05 + .30 + .15 = .50$$

C: {4 does not occur }

$$P(C) = P(1) + P(2) + P(3) + P(5)$$
$$= .05 + .20 + .30 + .15 = .70$$

PROBLEM # 3.10

Two marbles are drawn at random and without replacement from a box containing two blue marbles and three red marbles.

a. **List the sample points for this experiment**

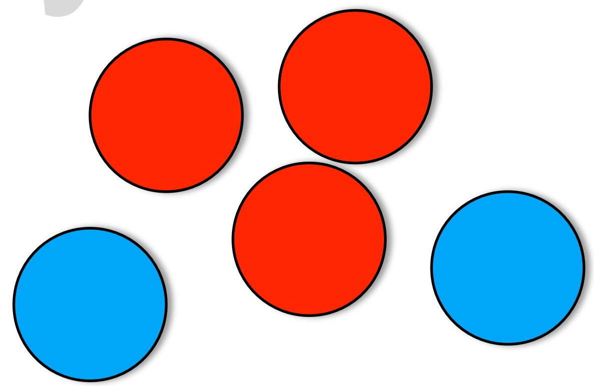
b. **Assign probabilities to the sample points**

c. **Determining the probability of observing each of the following events:**

A: { two blue marbles are drawn}

B: { A red and a blue marble are drawn}

C: { Two red marbles are drawn}

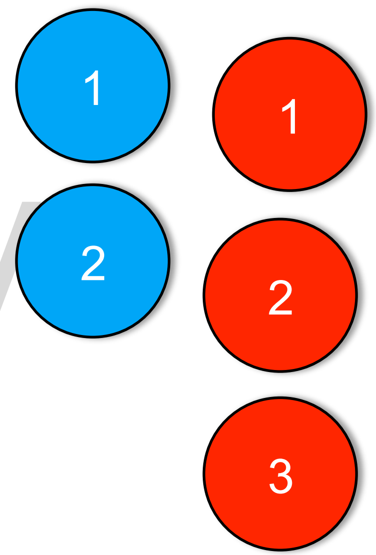


PROBLEM # 3.10

Two marbles are drawn at random and without replacement from a box containing two blue marbles and three red marbles.

a. List the sample points for this experiment

- **Let's call Blue Marbles : B_1, B_2**
- **Let's call red marbles: R_1, R_2, R_3**



There are 10 sample points:

(B_1, B_2) (B_1, R_1) (B_1, R_2) (B_1, R_3) (B_2, R_1)
(B_2, R_2) (B_2, R_3) (R_1, R_2) (R_1, R_3) (R_2, R_3)

PROBLEM # 3.10

Two marbles are drawn at random and without replacement from a box containing two blue marbles and three red marbles.

a. List the sample points for this experiment

b. **Assign probabilities to the sample points**

1/10

Each sample point are equally likely

PROBLEM # 3.10

Two marbles are drawn at random and without replacement from a box containing two blue marbles and three red marbles.

c. Determining the probability of observing each of the following events:

A: { two blue marbles are drawn} $A = \{(B_1, B_2)\}$. Thus, $\mathbf{P(A) = 1/10}$

B: { A red and a blue marble are drawn}

$B = \{ (B_1, R_1)\ (B_1, R_2)\ (B_1, R_3)\ (B_2, R_1)\ (B_2, R_2)\ (B_2, R_3)\}$.

Thus, $\mathbf{P(B) = 6\ (1/10) = 6/10 = 3/5}$

C: { Two red marbles are drawn}

$\mathbf{C} = \{(R_1, R_2)\ (R_1, R_3)\ (R_2, R_3)\}$ Thus, $\mathbf{P(C) = 3\ (1/10) = 3/10}$

PROBLEM # 3.10

A: { two blue marbles are drawn} $A = \{(B_1, B_2)\}$. Thus, **P(A)= 1/10**

$$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

On the first draw, there are 2 Blue marbles out of 5 marbles.
On the second draw there is 1 Blue marble left out of 4 marbles.

B: { A red and a blue marble are drawn}

$B = \{ (B_1, R_1)\ (B_1, R_2)\ (B_1, R_3)\ (B_2, R_1)\ (B_2, R_2)\ (B_2, R_3)\}$.

Thus, **P(B) = 6 (1/10) = 6/10 = 3/5**

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$

$$\frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$

$$\frac{12}{20} = \frac{6}{10}$$

C: { Two red marbles are drawn}

C $= \{(R_1, R_2)\ (R_1, R_3)\ (R_2, R_3)\}$ Thus, **P(C) = 3 (1/10) = 3/10**

$$\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

PROBLEM # 3.11

If there are 30 red and blue marbles in a jar, and the ratio of red to blue marbles is 2:3, what is the probability that, drawing twice, you will select two red marbles if you return the marbles after each draw?

4/25

Some Basic Probability Laws

Conditional Probability

BASIC RELATIONSHIPS OF PROBABILITY

COMPLEMENT OF AN EVENT

COMPLEMENT OF A

VENN DIAGRAM

ADDITION LAW

UNION OF TWO EVENTS

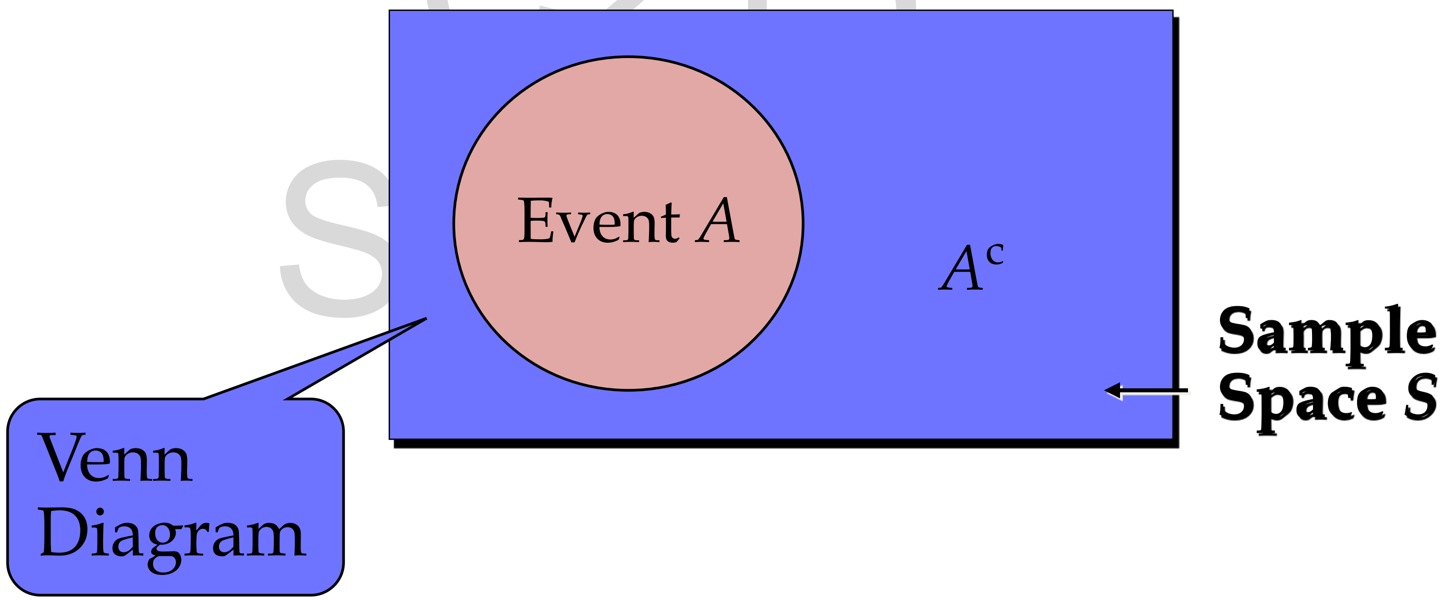
INTERSECTION OF TWO EVENTS

MUTUALLY EXCLUSIVE EVENTS

Bowerman, et al. (2017) pp. 229

COMPLEMENT OF AN EVENT

Complement of A is denoted as A^c



Bowerman, et al. (2017) pp. 229

COMPLEMENT OF AN EVENT

What is the probability of

a SAMPLE SPACE?

Event A: P(A) = 0.75.

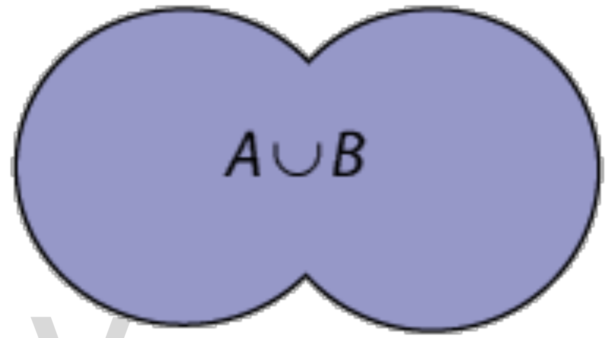
What is the probability of $P(A^c)$?

ADDITION LAW

Union of Two Events contains all sample points belonging to A or B or both.

“or” = Union “U”

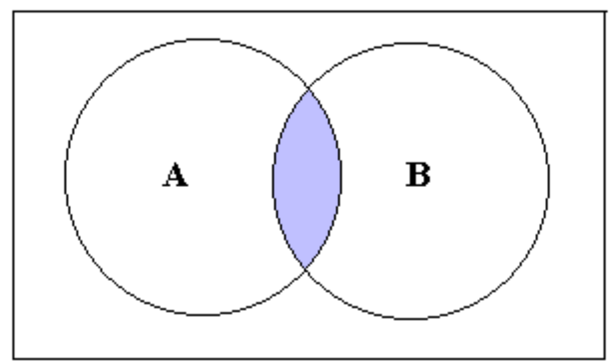
A U B



Intersection of Two Events contains the sample points belonging to both A and B.

“and” = Intersection “∩”

A ∩ B



Bowerman, et al. (2017) pp. 231

ADDITION LAW

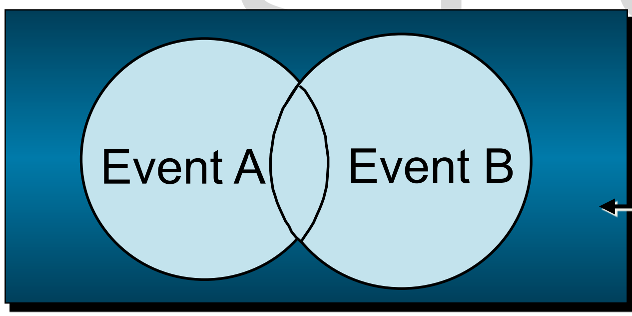
THE PROBABILITY THAT EVENT A OR EVENT B OR BOTH OCCUR

How can you explain Addition Law?

P(A) P(B) P(A∩ B)

-

$P(A \cup B) = \quad + $



Sample Space *S*

P(A∩ B)

Bowerman, et al. (2017) pp. 232

PROBLEM # 3.12

Survey on energy conservation. A state energy agency mailed questionnaires on energy conservation to 1,000 homeowners in the state capital. Five hundred questionnaires were returned. Suppose an experiment consists of randomly selecting and reviewing one of the returned questionnaires. Consider the events:

- A: {The home is constructed of brick}
- B: {The home is more than 30 years old}
- C: {The home is heated with oil}

Describe each of the following events in terms

of unions, intersections and complements

(ie., $A \cup B$, $A \cap B$, A^c, etc.):

PROBLEM #3.12

- A: {The home is constructed of brick}
- B: {The home is more than 30 years old}
- C: {The home is heated with oil}

Describe each of the following events in terms of unions, intersections and complements (ie., A U B, A ∩ B, A^c , etc.):

The home is more than 30 years old and is heated with oil.

ANSWER: $B \cap C$ $P(B \cap C)$

The home is not constructed of brick

ANSWER: A^c A' A^c $P(\bar{A})$

PROBLEM #3.12

- A: {The home is constructed of brick}
- B: {The home is more than 30 years old}
- C: {The home is heated with oil}

Describe each of the following events in terms of unions, intersections and complements (ie., A U B, A ∩ B, A^c, etc.):

The home is heated with oil or is more than 30 years old.

ANSWER: C U B P(B U C)

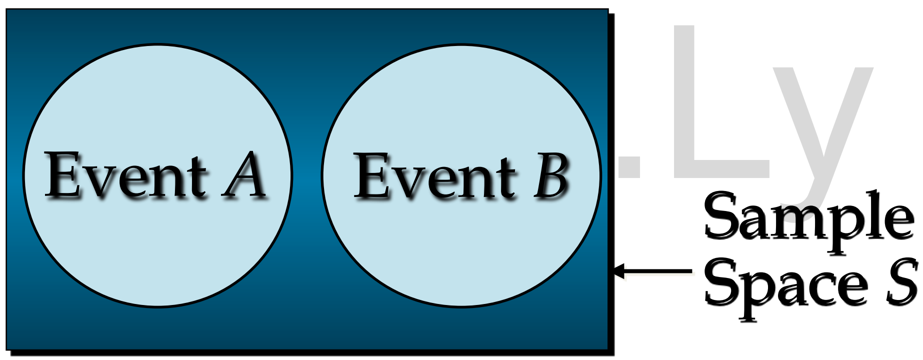
The home is constructed of brick and is not heated with oil.

ANSWER: A ∩ C^c P(A ∩ C')

MUTUALLY EXCLUSIVE EVENTS

IF THE EVENTS HAVE NO SAMPLE POINTS IN COMMON.

$$P(A \cup B) = P(A) + P(B)$$



Nothing!

$P(A \cap B)$

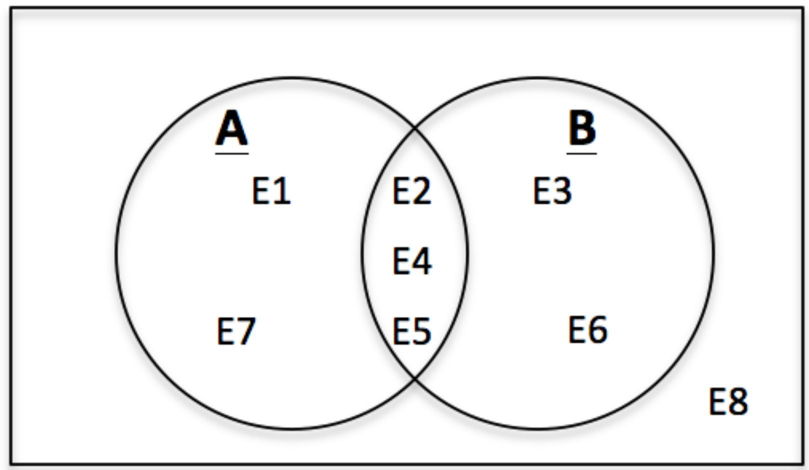
Bowerman, et al. (2017) pp. 233

PROBLEM #3.13

Draw the Venn diagram where

$P(E_1) = .10$, $P(E_2) = .05$, $P(E_3) = P(E_4) = .2$, $P(E_5) = .06$, $P(E_6) = .3$, $P(E_7) = .06$ and $P(E_8) = .03$.

Find the following probabilities:



$P(A^c)$

$P(B^c)$

$P(A^c \cap B)$

$P(A \cup B)$

$P(A \cap B)$

$P(A^c \cap B^c)$

Are events A and B mutually exclusive? Why?

PROBLEM #3.13

Find the following probabilities:

$P(E_1) = .10$ | $P(E_5) = .06$
$P(E_2) = .05$ | $P(E_6) = .3$
$P(E_3) = .2$ | $P(E_7) = .06$
$P(E_4) = .2$ | $P(E_8) = .03$

$P(A^c)$ $= E3+E6+E8$

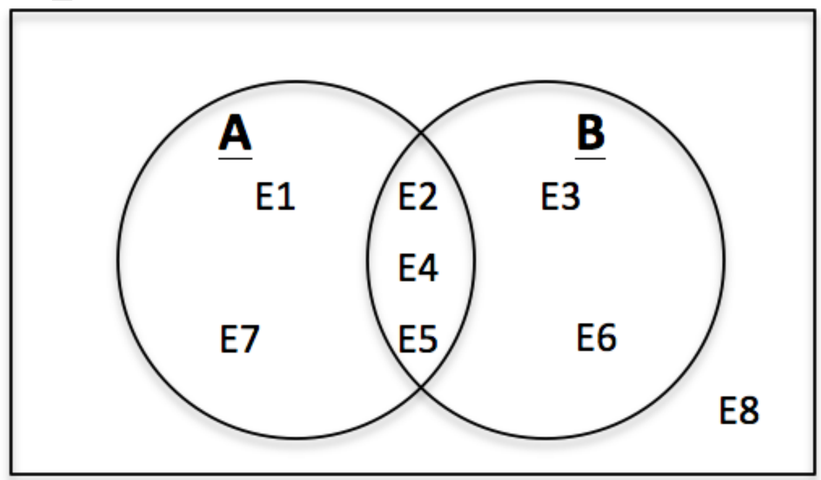
$= 0.2+0.3+0.03 = 0.53$

$P(B^c)$ $= E1+E7+E8$

$= 0.10 +0.06 +0.03 = 0.19$

$P(A^c \cap B)$ $= E3+E6$

$= 0.2 +0.3= 0.5$



PROBLEM #3.13

$P(E_1) = 0.10$ $P(E_5) = 0.06$

$P(E_2) = 0.05$ $P(E_6) = 0.3$

$P(E_3) = 0.2$ $P(E_7) = 0.06$

$P(E_4) = 0.2$ $P(E_8) = 0.03$

Find the following probabilities:

P(A U B) = E1+E2+E3+E4+E5+E6+E7

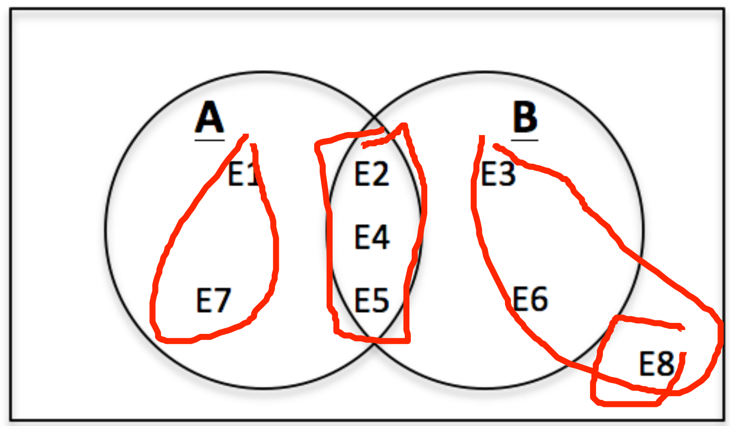
or..= 1-E8

= 0.1+0.05 +0.2 + 0.2 + 0.06 + 0.3 + 0.06 = 0.97

P(A ∩ B) = E2+E4+E5

= 0.05 +0.2 + 0.06 = 0.31

$P(A^c \cap B^c)$ = E8 = 0.03



Are events A and B mutually exclusive? Why?

Conditional Probability

CONDITIONAL PROBABILITY

CONDITIONAL PROBABILITY

JOINT PROBABILITIES

MARGINAL PROBABILITIES

INDEPENDENT EVENTS

MULTIPLICATION LAW

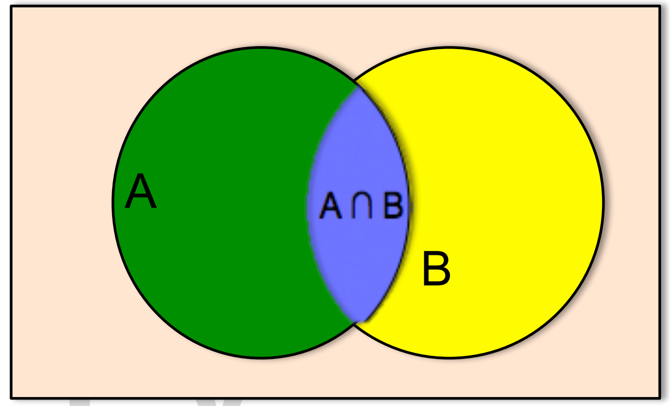
MULTIPLICATION LAW FOR INDEPENDENT EVENTS

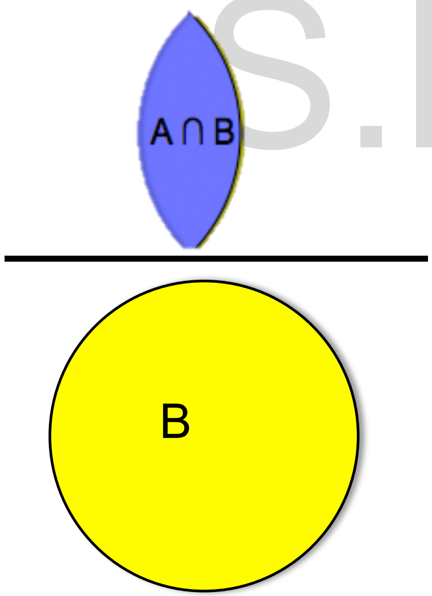
Bowerman, et al. (2017) pp. 235

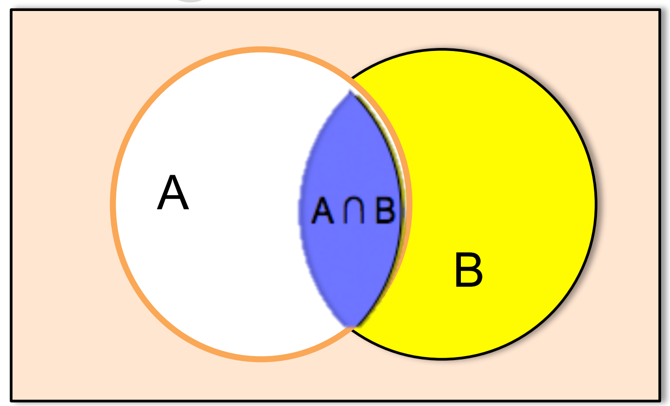
CONDITIONAL PROBABILITY

A GIVEN B P(A|B)

$$P(A \mid B) = \frac{P(A \cap B)}{\boxed{P(B)}}$$



P(A|B) = 



Bowerman, et al. (2017) pp. 235

PROBLEM # 3.14

Given that P(A∩B) =.4 and P(A|B) = .8, find P(B)

Since the P(A|B) = P(A∩B)/P(B), substitute the given probabilities into the formula and solve for P(B).

P(A|B) = $.8 = \frac{.4}{P(B)} = \mathrm{P(B)} = \frac{.4}{.8} = .5$

CONDITIONAL PROBABILITY
PROBLEM # 3.15

Joint probability- intersection of two events

	A_1	A_2	A_3	
B_1	.15	.20	.10	$P(B_1)$ = .45
B_2	.25	.25	.05	$P(B_2)$ = .55
	$P(A_1)$ = .40	$P(A_2)$ = .45	$P(A_3)$ = .15	1

Marginal probability- located in the margins of the joint probability table

PROBLEM # 3.15, 3.16, 3.17

	A_1	A_2	A_3	
B_1	.15	.20	.10	$P(B_1) = .45$
B_2	.25	.25	.05	$P(B_2) = .55$
	$P(A_1) = .40$	$P(A_2) = .45$	$P(A_3) = .15$	

3.16 a. Compute $P(A_2|B_2)$

$$\frac{P(A_2 \text{ and } B_2)}{P(B_2)} = \frac{.25}{.55} = .455$$

3.16 c. Compute $P(B_1|A_2)$

$$\frac{P(A_2 \text{ and } B_1)}{P(A_2)} = \frac{.20}{.45} = .444$$

PROBLEM # 3.15, 3.16, 3.17

	A_1	A_2	A_3	
B_1	.15	.20	.10	$P(B_1)$ =.45
B_2	.25	.25	.05	$P(B_2)$ =.55
	$P(A_1)$ =.40	$P(A_2)$ =.45	$P(A_3)$ =.15	

3.17 a. Compute P(A_1 or A_2) $P(A_1 \cup A_2)$

$$= P(A_1) + P(A_2) = .40 + .45 = .85$$

3.17 b. Compute P(A_2 or B_2)

$$= P(A_2) + P(B_2) - P(A_2 \text{ and } B_2) = .45 + .55 - .25 = .75$$

INDEPENDENT EVENTS

When Event A is affected by Event B, then

- Event A and B dependent

When the probability of Event A is not changed by the existence of Event B

- Event A and B are independent

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

Bowerman, et al. (2017) pp. 238

PROBLEM # 3.18

A firm has classified its customers in two ways: (1) according to whether the account is overdue and (2) whether the account is new (less than 12 months) or old. An analysis of the firm's records provided the input for the following table of joint probabilities.

	Overdue	Not Overdue	Total
New	.06	.13	.19
Old	.52	.29	.81
Total	.58	.42	

- **a. If the account is overdue, what is the probability that it is new?**

P(new | overdue) = $$\frac{P(new\ and\ overdue)}{P(overdue)} = \frac{.06}{.06+.52} = \frac{.06}{.58} = .103$$

PROBLEM # 3.18

	Overdue	Not Overdue	Total
New	.06	.13	.19
Old	.52	.29	.81
Total	.58	.42	

A firm has classified its customers in two ways: (1) according to whether the account is overdue and (2) whether the account is new (less than 12 months) or old. An analysis of the firm's records provided the input for the following table of joint probabilities

- **b. If the account is new, what is the probability that it is overdue?**

P(overdue | new) = $\frac{P(new\ and\ overdue)}{P(new)} = \frac{.06}{.06 + .13} = \frac{.06}{.19} = .316$

- **c. Is the age of the account related to whether it is overdue? Explain.**

Yes, because P(new) = .19 ≠ P(new| overdue)

MULTIPLICATION LAW

The ADDITION LAW is used to compute the probability of a <u>union of two events</u>.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The MULTIPLICATION LAW is used to compute the probability of an <u>intersection of two events.</u>

Based on CONDITIONAL PROBABILITY.

$$P(A \cap B) = P(B)P(A \mid B)$$

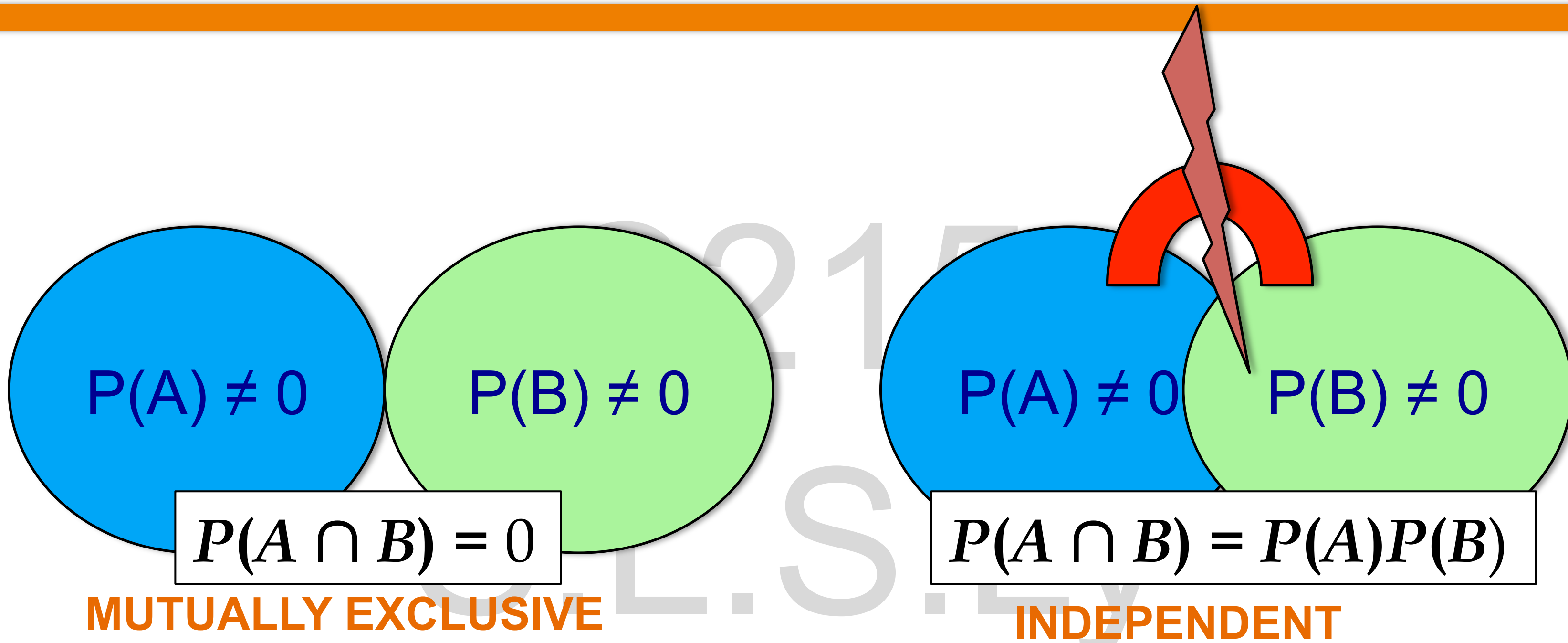
$P(A|B) = P(A)$

FOR INDEPENDENT EVENTS:

$P(A \cap B) = P(A)P(B)$

$$P(A \cap B) = P(A)P(B)$$

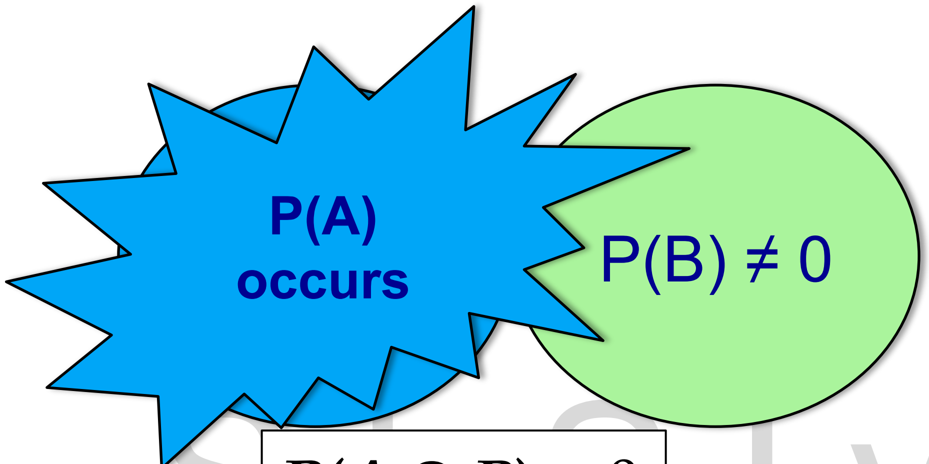
MUTUALLY EXCLUSIVE VS INDEPENDENT



CANNOT HAPPEN AT THE SAME TIME

- **Two events with nonzero probabilities cannot be both mutually exclusive and independent.**

MUTUALLY EXCLUSIVE VS INDEPENDENT

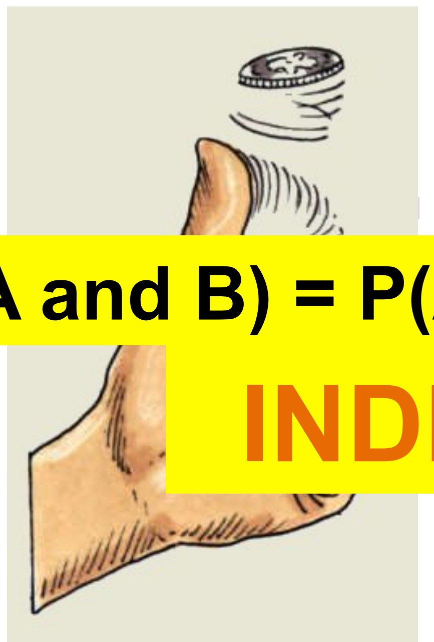


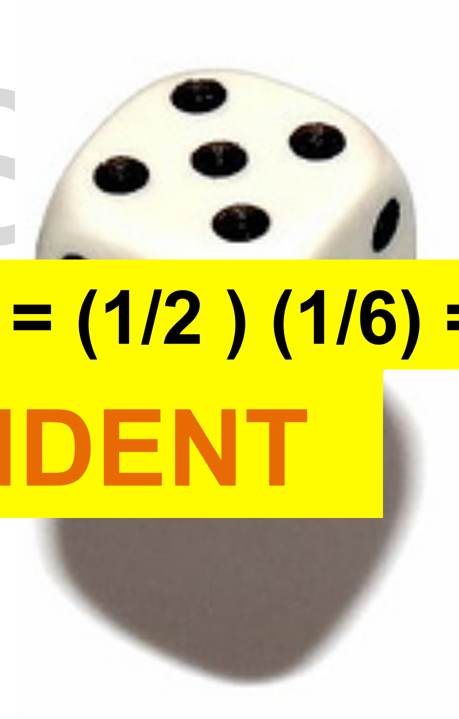
$$P(A \cap B) = 0$$

MUTUALLY EXCLUSIVE

- **If one mutually exclusive event is known to occur, the other cannot occur.; thus, the probability of the other event occurring is reduced to zero (and they are therefore dependent).**
- **Two event that are not mutually exclusive, might or might not be independent.**

Event A : { Head }   Event B : { 6 }





P (A and B) = P(A) P(B) = (1/2) (1/6) = 1/12

INDEPENDENT

Event A : { Blue Face } **Event B : { 3 }**

MUTUALLY EXCLUSIVE

P (A and B) = 0

P(A) = 1/2

P(B) = 1/6

IF THEY ARE MUTUALLY EXCLUSIVE, THEN THEY ARE DEPENDENT

PROBLEM #3.19

A and B are mutually exclusive events, with P(A) = 0.2 and P(B) = 0.3.

a. Find P(A|B)

If events A and B are mutually exclusive, then $P(A \cap B) = 0$.

$P(A|B) = P(A \cap B) / P(B) = 0/0.3 = 0$

b. Are A and B independent events?

If the events A and B are independent, then $P(A|B) = P(A)$.

$P(A|B) = 0$, $P(A) = .2$

$$0 \neq .2$$

Thus, events A and B are not independent.

PROBLEM # 3.21

An investor believes that on a day when the Dow Jones industrial Average (DJIA) increase, the probability that the NASDAQ also increases is 77%. If the investor believes that there is a 60% probability that the DJIA will increase tomorrow, what is the probability that the NASDAQ will increase as well?

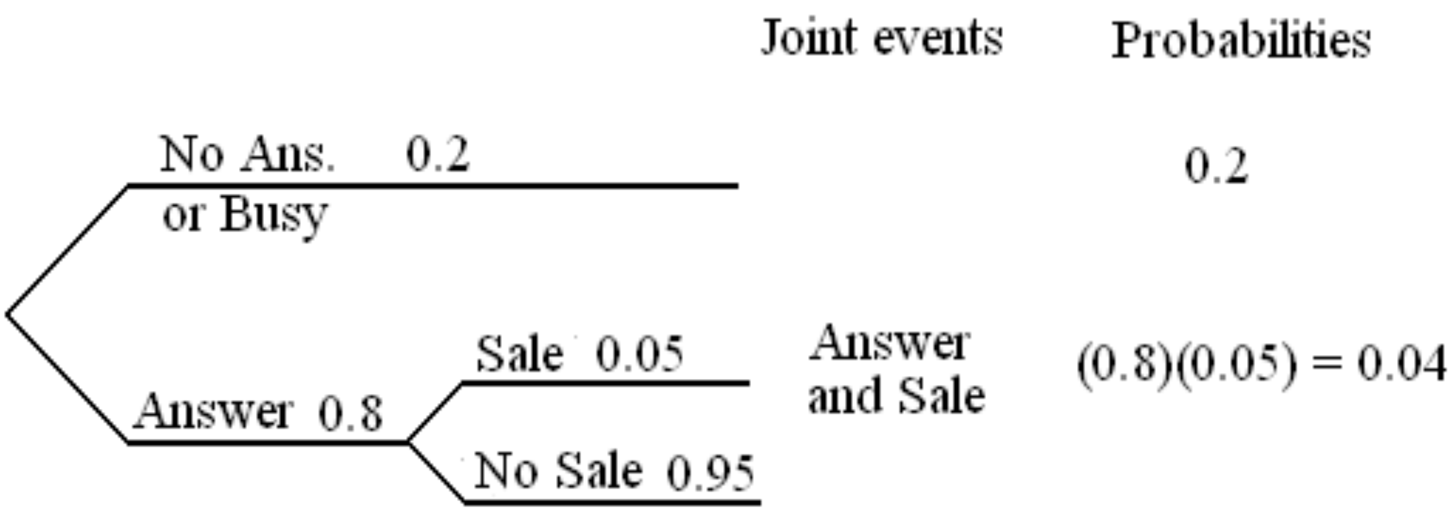
Let A = DJIA increase and B = NASDAQ increase

$P(A) = .60$ and $P(B \mid A) = .77$

$P(A \text{ and } B) = P(A)P(B \mid A) = (.60)(.77) = .462$

PROBLEM # 3.2

A telemarketer calls people and tries to sell them a subscription to a daily newspaper. On 20% of her calls, there is no answer or the line is busy. She sells subscriptions to 5% of the remaining calls. For what proportion of calls does she make a sale?



Assume you have applied to two different universities (let's refer to them as Universities A and B) for your graduate work. In the past, 25% of students (with similar credentials as yours) who applied to University A were accepted, while University B accepted 35% of the applicants. Assume events are independent of each other.*

What is the probability that you will be accepted to at least one graduate program?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.25 + .35 - (.25)(.35)$

$P(A \cap B) = P(A) \times P(B)$ $= 0.5125$