



COMM215

First the Foundation, then Innovation

LESSON TWO

DESCRIPTIVE STATISTICS

SAMIE L.S. LY

Qualitative and Quantitative Data

Describe Central Tendency

Measures of Variation

Percentiles, Quartiles, and Box and Whiskers.

Midterm Preparation



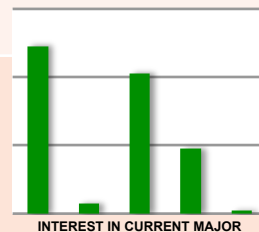
QUALITATIVE DATA

QUANTITATIVE DATA

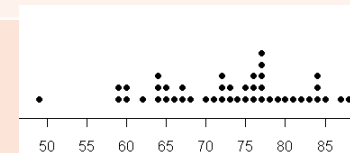
FREQUENCY DISTRIBUTION

RELATIVE FREQUENCY PERCENT FREQUENCY DISTRIBUTION

BAR CHART



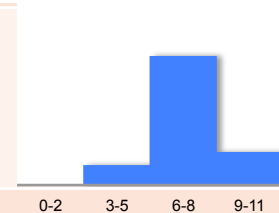
DOT PLOT



PIE CHART



HISTOGRAM



CUMULATIVE DISTRIBUTION

DESCRIBING QUALITATIVE DATA



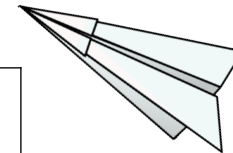
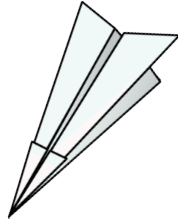
- FREQUENCY DISTRIBUTION
- RELATIVE FREQUENCY DISTRIBUTION
- PERCENT FREQUENCY DISTRIBUTION
- BAR CHARTS, PIE CHARTS

INTEREST IN YOUR MAJOR?



FREQUENCY DISTRIBUTION is

a tabular summary of data showing the number of items in each of several non-overlapping classes.



INTEREST IN MAJOR	FREQUENCY
WELL-PAYED JOB	49
PROMOTION	3
GRAD SCHOOL	41
NO CLUE	19
FREE	1
	113

RELATIVE FREQUENCY



FREQUENCY ÷ TOTAL N

41 ÷ 113

= 0.363

INTEREST IN MAJOR	FREQUENCY	RELATIVE FREQUENCY
WELL-PAYED JOB	49	
PROMOTION	3	0.027
GRAD SCHOOL	41	0.363
NO CLUE	19	
FREE	1	0.009
TOTAL	113	

RELATIVE FREQUENCY



FREQUENCY $\div$ TOTAL N

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INTEREST IN YOUR MAJOR?



PERCENT FREQUENCY: RELATIVE FREQUENCY X 100

INTEREST IN MAJOR	FREQUENCY	RELATIVE FREQUENCY	PERCENT FREQUENCY
WELL-PAYED JOB	49	0.434	43.363
PROMOTION	3	0.027	2.655
GRAD SCHOOL	41	0.363	
NO CLUE	19	0.168	16.814
FREE	1	0.009	0.885
TOTAL	113	1.000	100.000

INTEREST IN YOUR MAJOR?



PERCENT FREQUENCY: RELATIVE FREQUENCY X 100

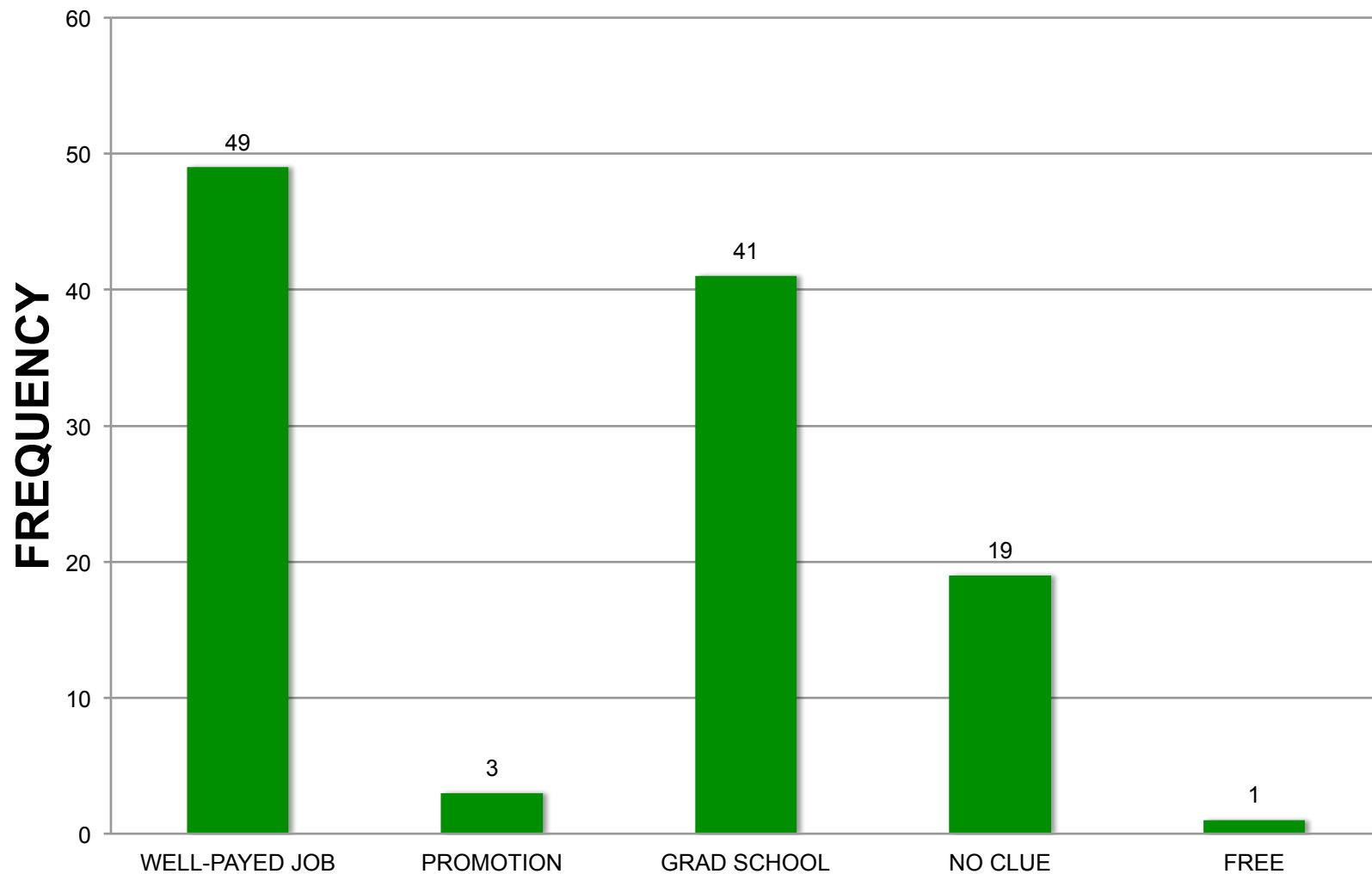
$$0.434 \times 100 = 43.363$$

INTEREST IN MAJOR	FREQUENCY	RELATIVE FREQUENCY	PERCENT FREQUENCY
WELL-PAYED JOB	49	0.434	43.363
PROMOTION	3	0.027	2.655
GRAD SCHOOL	41	0.363	
NO CLUE	19	0.168	16.814
FREE	1	0.009	0.885
TOTAL	113	1.000	100.000

BAR GRAPH



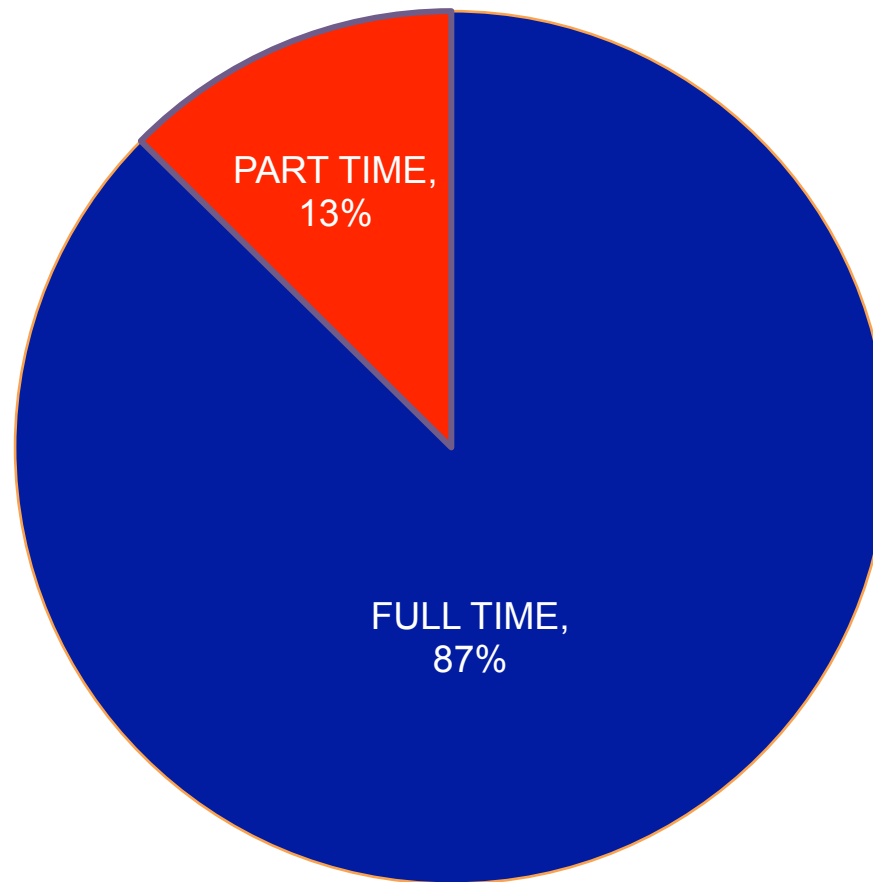
Sample Size: **120 STUDENTS**



INTEREST IN CURRENT MAJOR

Bowerman, et al. (2017) pp. 56

UNIVERSITY STATUS

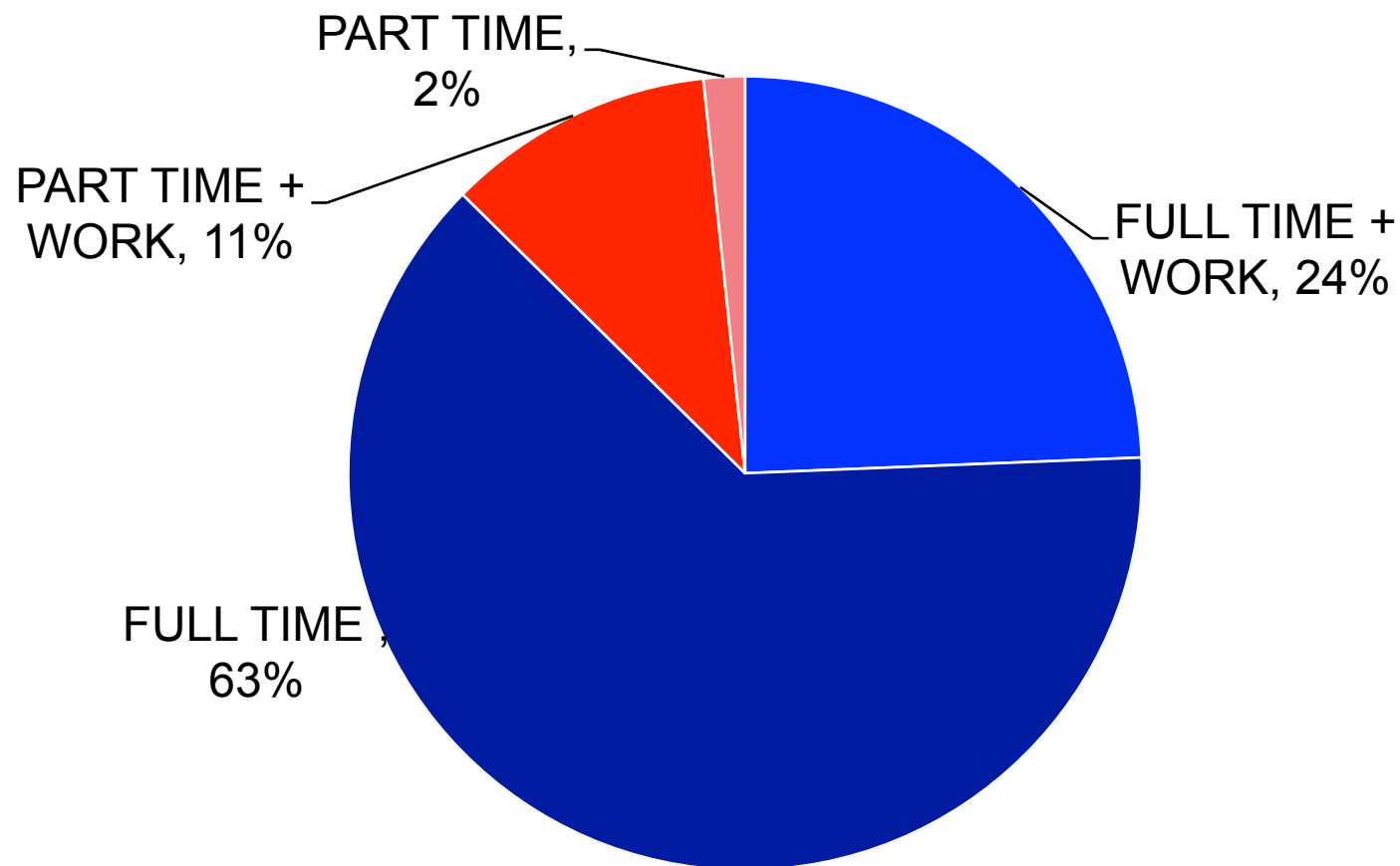


Bowerman, et al. (2017) pp. 58

UNIVERSITY STATUS + WORK



Sample size: **120** STUDENTS



Bowerman, et al. (2017) pp. 58

SUMMARIZING QUALITATIVE DATA



Qualitative Data

- Grouped by specific categories.
- Used either the NOMINAL scale or the ORDINAL scale of measurement.
- NOMINAL: 1: Male, 2: Female
- ORDINAL: Excellent, Very Good, Good, Poor

SUMMARIZING QUANTITATIVE DATA



- **FREQUENCY DISTRIBUTION**
 - Relative Frequency
 - Percent Frequency
- **CUMULATIVE DISTRIBUTION**
 - Cumulative Relative Frequency
 - Cumulative Percent Frequency
- **HISTOGRAM**

BUILDING A HISTOGRAM



0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20

NUMBER OF CLASSES (K)

Possible hours of sleep in a day: 0 to 20.

*RECOMMENDED TO USE 5 TO 20 CLASSES

GOAL: use enough classes to show a variation in data

[	]	[	]
[	]	[	]
[	]	[	]
		[	]

WIDTH OF CLASSES

[LARGEST VALUE-SMALLEST VALUE]

NUMBER OF CLASSES

$$\frac{20 - 0}{7} = 2.85$$

Bring this number to the next integer

$$2^K > n$$

SLEEP TIME



NUMBER OF CLASSES

I CHOSE TO HAVE 7 CLASSES,

[0-2], [3-5], [6-8], [9-11] , [12-14], [15-17],[18-20]

CLASS LIMITS

Each Item belongs to one class and not the other.

2 HOURS OF SLEEP FALLS INTO [0-2]

3 HOURS OF SLEEP FALLS INTO [3-5]

SLEEP TIME



$$1+8+93 = 102$$

SLEEP TIME PER DAY	FREQUENCY	RELATIVE FREQUENCY	PERCENT FREQUENCY
0-2	1	0.008	0.847
3-5	8	0.068	6.780
6-8	93	0.788	78.814
9-11	16	0.136	13.559
TOTAL	118	1	100

SLEEP TIME PER DAY	CUMULATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY	CUMULATIVE PERCENT FREQUENCY
LESS THAN OR EQUAL TO 2	1	0.008	0.847
LESS THAN OR EQUAL TO 5	9		7.627
LESS THAN OR EQUAL TO 8	118		
LESS THAN OR EQUAL TO 11		1.000	100.000

SLEEP TIME



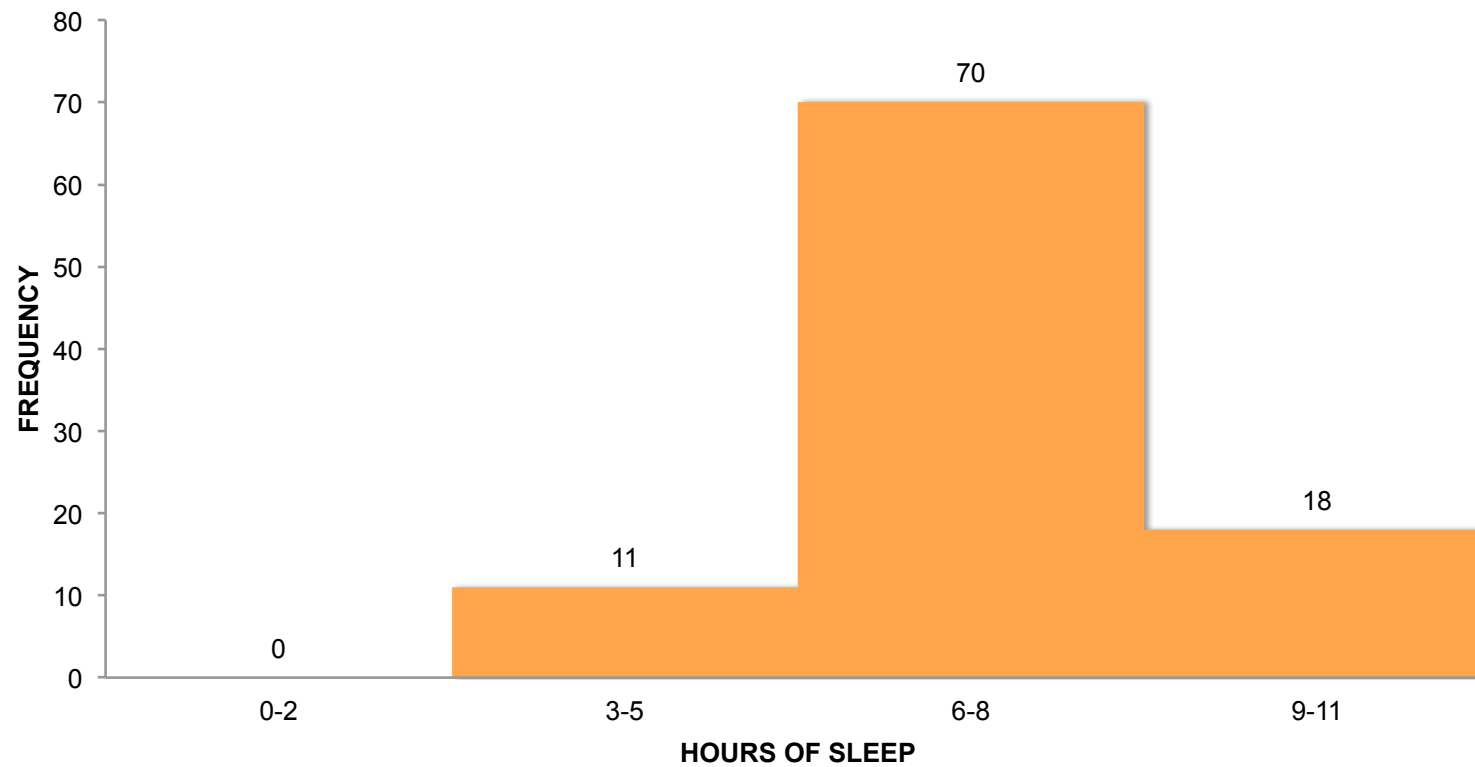
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SLEEP TIME

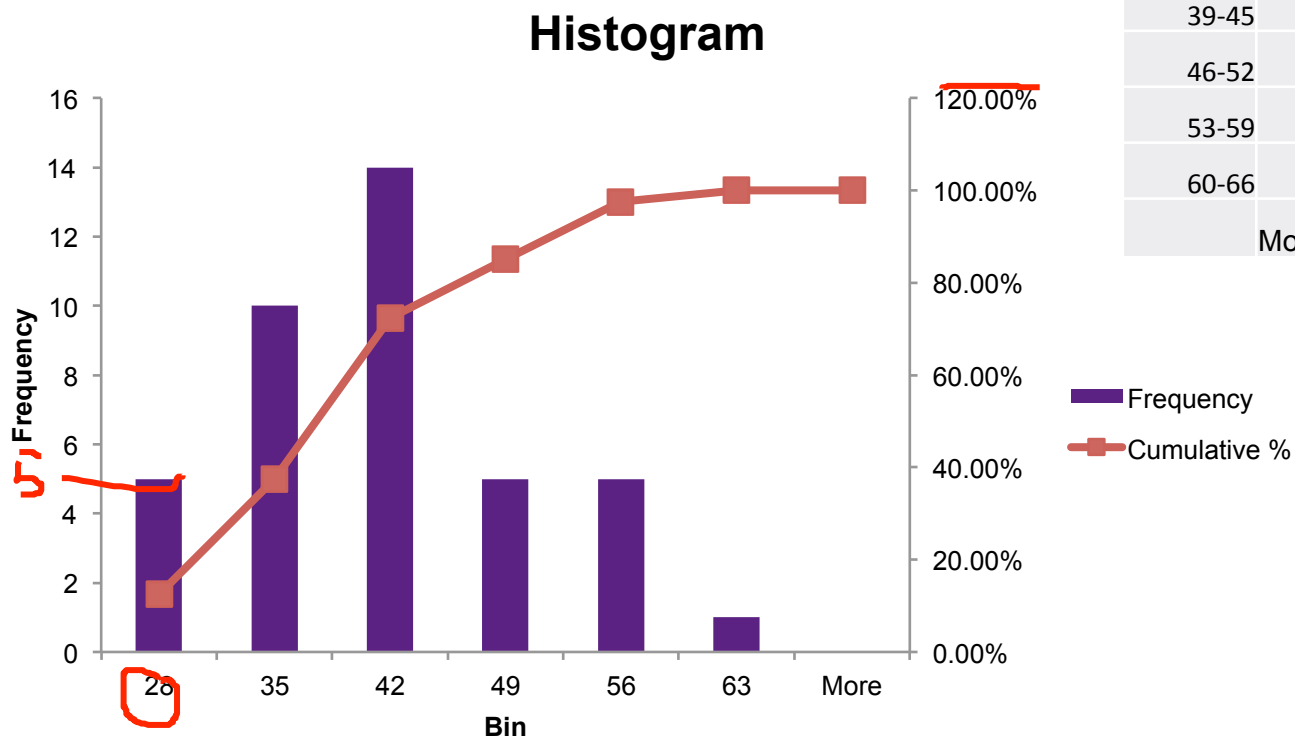


HISTOGRAM FOR SLEEP TIME





A graph of a cumulative distribution



Width	Midpoint	Frequency	Cumulative %
25-31	28	5	12.50%
32-38	35	10	37.50%
39-45	42	14	72.50%
46-52	49	5	85.00%
53-59	56	5	97.50%
60-66	63	1	100.00%
More		0	100.00%

STEM AND LEAF DISPLAY



A way to summarize number distributions

72	48	
54	59	
9	50	
9	51	
9	52	6
321	53	7
986	54	5
77755443221	55	556
977	56	024789
96220	57	23466679
766544422211000	58	00034444556678889
99988877643333210	59	01223355567788899
743211110	60	00012233444455667789
887655544322110	61	22224444555566777789
999988875544322110	62	00112223334444667777889999
98665543333321000	63	00112234555668889999
988887766554333221110	64	000001122245566689
985544200	65	12334677789
99866422220	66	00478
77664	67	00136
9331	68	8
8210	69	
	70	
9	71	
	72	
3	73	



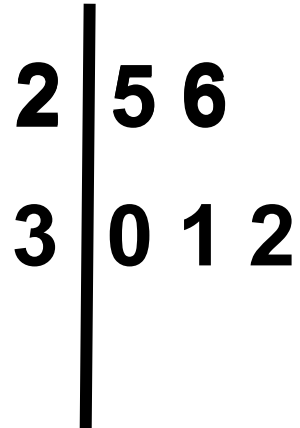


DATA SET:
25, 26, 30, 31, 32

STEM AND LEAF DISPLAY



LEAF UNIT



LEAF UNIT = 1

DATA SET: 25 x 1 26 x 1

LEAF UNIT = 10

25 x 10 26 x 10

DATA SET: 250, 260

LEAF UNIT = 0.1

25 x 0.1 26 x 0.1

DATA SET: 2.5, 2.6

PROBLEM # 2.1

A large investment firm on Wall Street wants to review the distribution of ages of its stockbrokers. The firm believes that this information can be useful in developing plans to recruit new brokers. The ages of a sample of 40 brokers are shown here.

46	28	51	34	29	40	38	33	41	52
53	40	50	33	36	41	-25	38	37	41
36	50	46	33	-61	48	32	28	30	49
41	37	26	39	35	39	46	26	31	35

- Draw a stem and leaf display
- Draw a histogram

25

LEAF UNIT



DATA SET

Sometimes...You will have to cut the numbers out.

STEM & LEAF DISPLAY

DATA SET: 356, 451, 423

LEAF UNIT: 10

3|6

4|2 5

Round the data!
356 becomes 360

HISTOGRAM PROBLEM # 2.1

NUMBER OF CLASSES

Largest value = 61 ,
Smallest value = 25

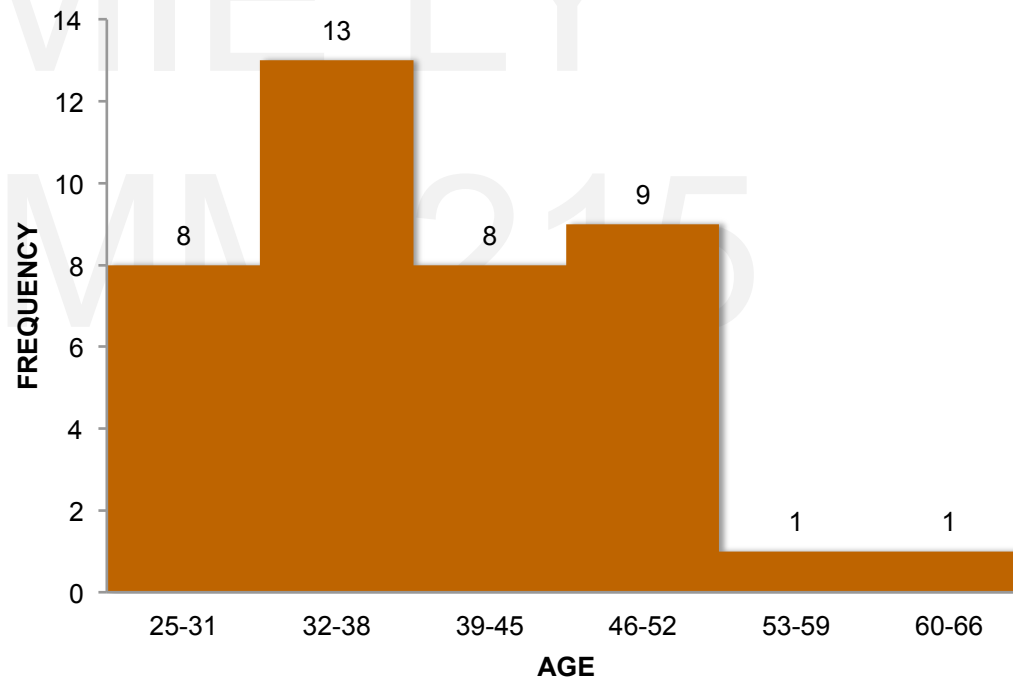
6 classes

$$[61-25]/6 = 6$$

CLASS WIDTH

[25-31], [32-38],
[39-45], [46-52],
[53-59], [60-66]

HISTOGRAM FOR AGE OF
STOCKBROKERS



PROBLEM # 2.2

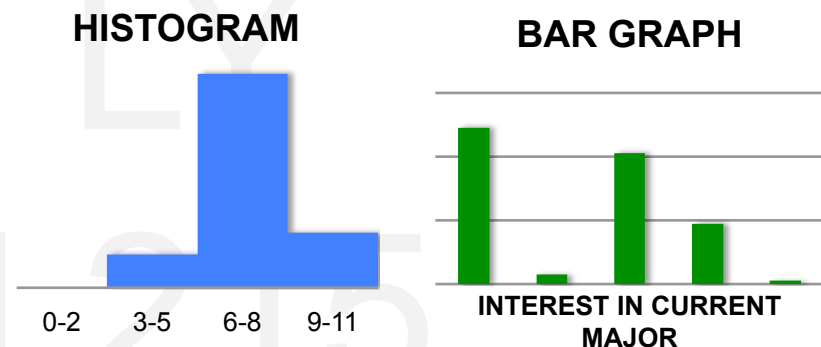
What is the difference between a histogram ^{and} a bar chart?
For what type of data would each be appropriate?

A histogram graphically displays

- class intervals
- class frequencies.

A bar chart displays

- frequencies for a set of categories or classes.



Histograms are appropriate for quantitative data

Bar charts are better for qualitative data.

Describe Central Tendency

Measures of Variation

Percentiles, Quartiles, and Box and Whiskers.

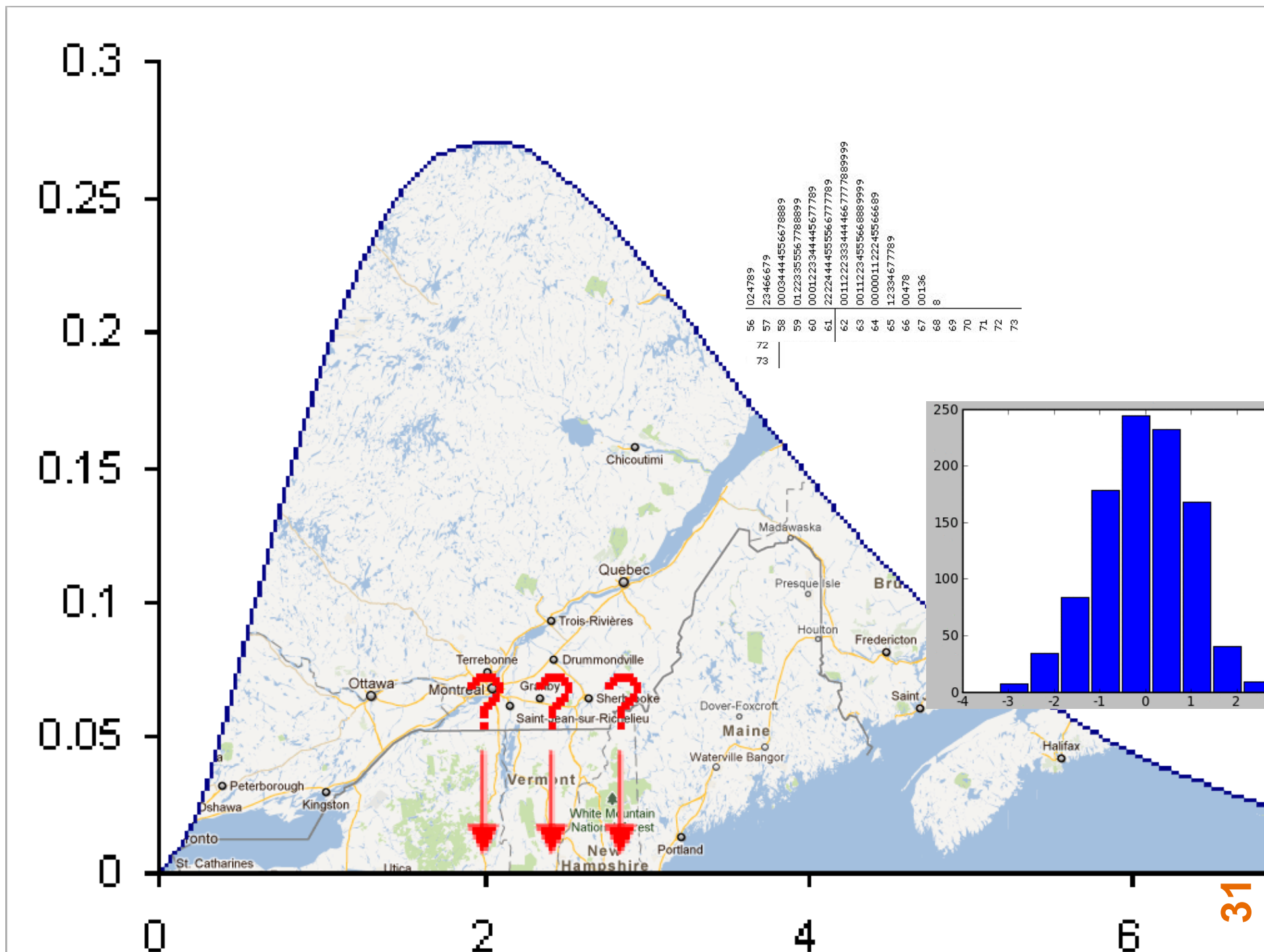
Midterm Preparation



MONTREAL

Where is Montreal Located?

How far from other cities is Montreal?



MEASURES OF LOCATION

- MEAN
- MEDIAN
- MODE

PROBLEM # 2.5

Consider the following sample of $n = 7$

measurements: $\boxed{5}, 7, 4, \boxed{5}, 20, 6, 2$.

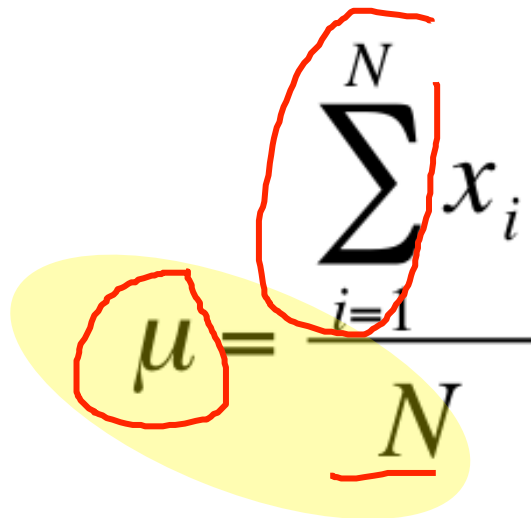
- Calculate the mean of this sample.
- Calculate the median of this sample.
- Eliminate the last measurement (2) and calculate the median, $n = 6$
- What is the mode?

$$\frac{5+6}{2} = 5.5$$

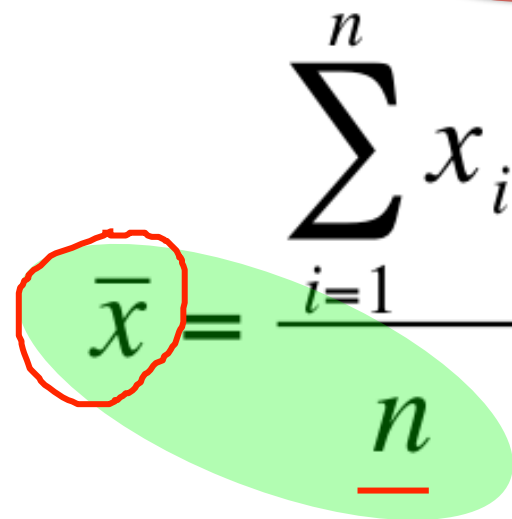
~~2~~, 4, 5, 5, 6, 7, 20

MEAN...(AVERAGE)

EXTREME
numbers can
inflate the mean.


$$\mu = \frac{\sum_{i=1}^N x_i}{N}$$

Population Mean


$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Sample Mean

PROBLEM # 2.5

Consider the following sample of $n = 7$

measurements: 5, 7, 4, 5, 20, 6, 2.

- a. Calculate the mean of this sample.

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{5 + 7 + 4 + 5 + 20 + 6 + 2}{7} = 7$$

ARRANGE NUMBERS in ORDER

MEDIAN- MIDDLE VALUE

ODD NUMBER OF N

Median is the middle value

$$n = 5$$

$$5/2 = 2.5$$

The median will be the 3rd value.

ROUND UP!

EVEN NUMBER OF N

Median is the average of the two middle values

$$n = 6$$

$$6/2 = 3$$

Take the 3rd and 4th value and make an average.

PROBLEM # 2.5

Consider the following sample of $n = 7$

measurements: 5, 7, 4, 5, 20, 6, 2.

- b. Calculate the median of this sample.
- c. Eliminate the last measurement (2) and calculate the median, $n = 6$

2, 4, 5, 5, 6, 7, 20

Median's position:

$7/2 = 3.5$ ROUND UP

at the 4th position

Median (m) = 5

4, 5, 5, 6, 7, 20

Median's position:

$6/2 = 3$

Average of 3rd and 4th position.

Median (m) = $(5+6)/2 = 5.5$

PROBLEM # 2.5

Consider the following sample of $n = 7$

measurements: 5, 7, 4, 5, 20, 6, 2.

d. What is the mode?

MODE: The Most Repeated Number in your sample

Mode = 5

NEW Data Set: 5, 7, 4, 5, 20, 6, 2, 7

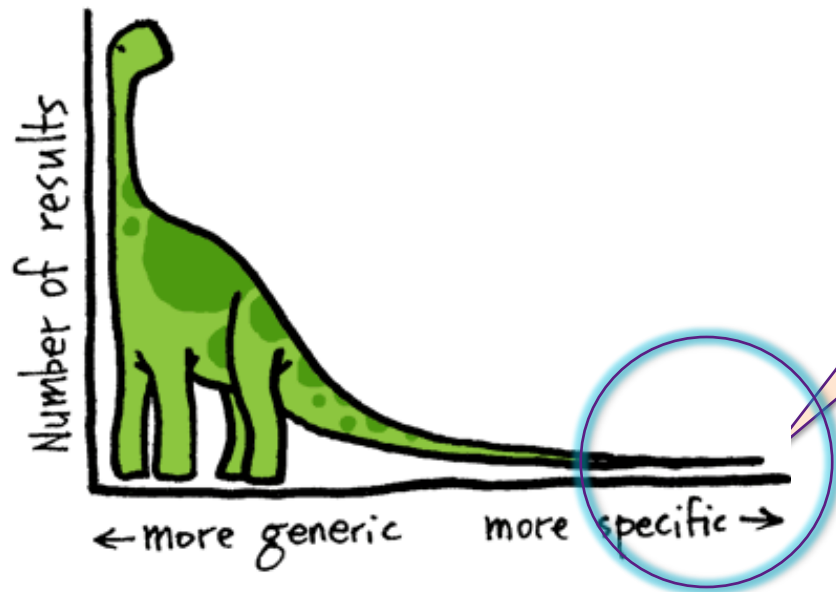
Which are the modes? 5 & 7

SHAPES OF HISTOGRAMS...

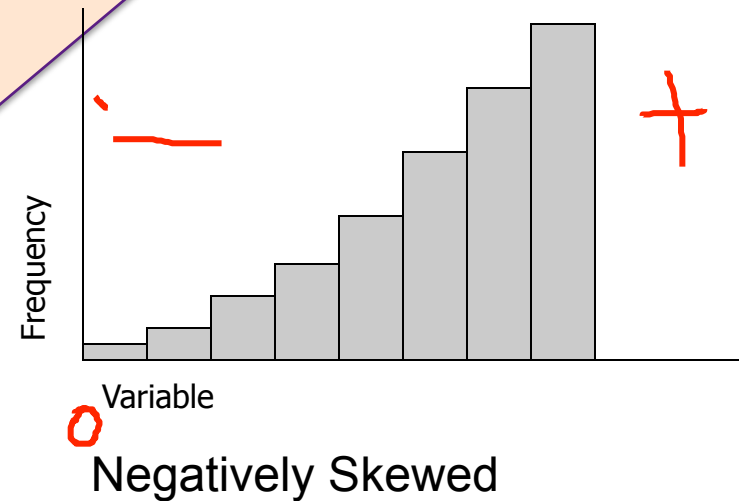


Skewness

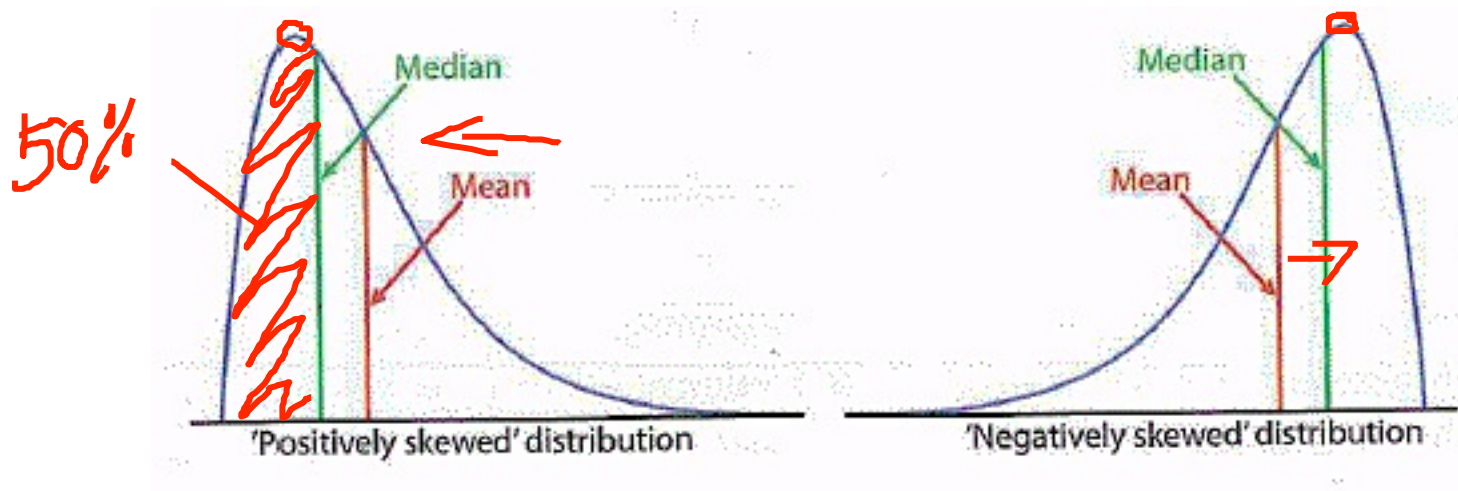
A skewed histogram is one with a long tail extending to either the right or the left:



WHERE THE TAIL GOES,
CONSIDER IT THE SIDE.



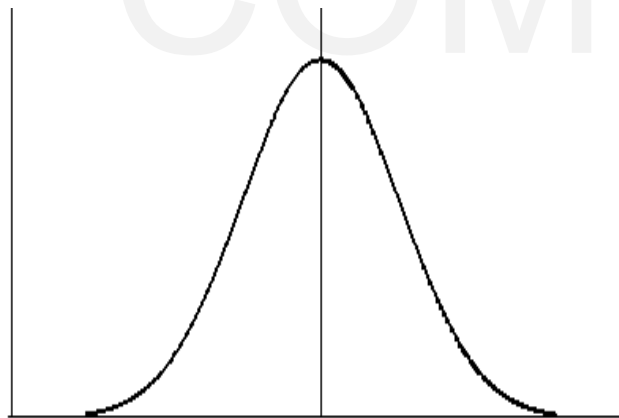
SKEWNESS



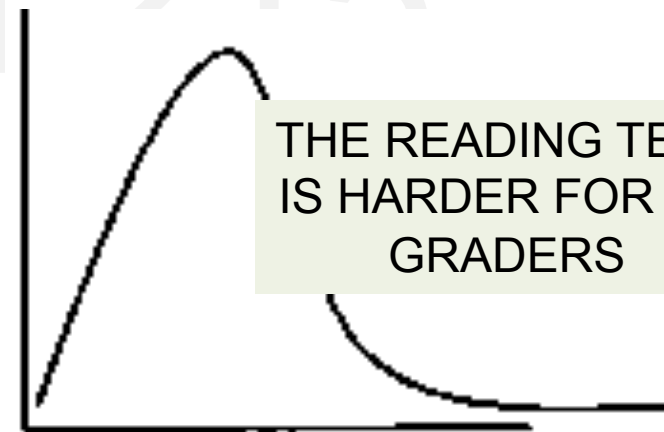
PROBLEM # 2.7

A reading test with 120 possible points yields a bell-shaped distribution with scores ranging from 5 to 50 on a large sample of third graders. If the same test were administered to first graders, what would we expect the form of the frequency distribution to be?

3RD GRADERS



1ST GRADERS



THE READING TEST
IS HARDER FOR 1ST
GRADERS

Measures of Variation

Percentiles, Quartiles, and Box and Whiskers.

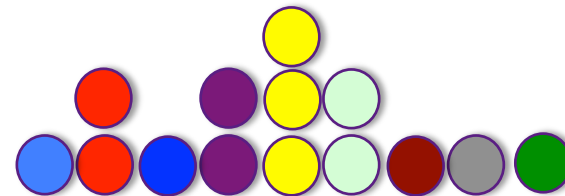
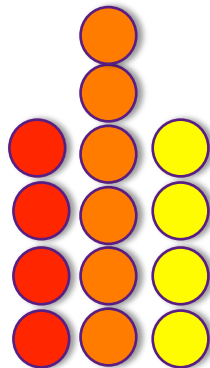
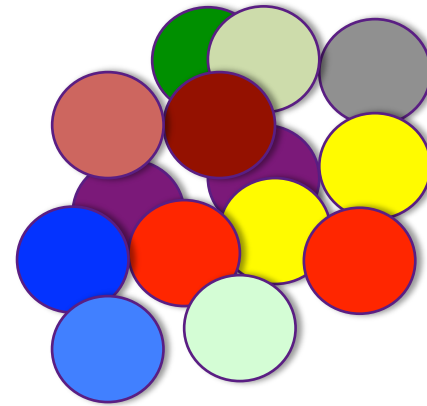
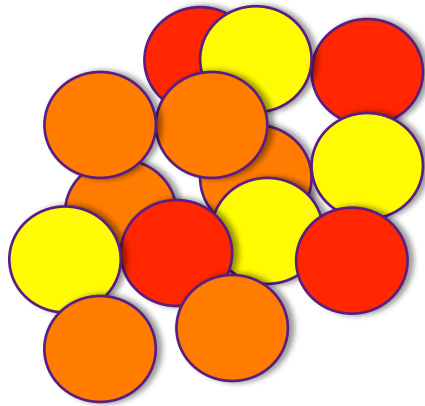
Midterm Preparation

MEASURES OF VARIATION



- RANGE
- VARIANCE
- STANDARD DEVIATION
- COEFFICIENT OF VARIATION

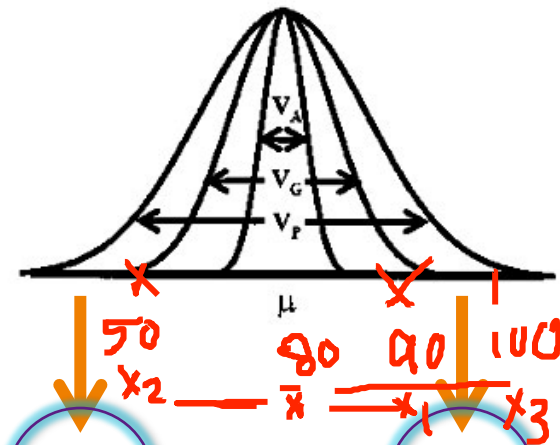
VARIABILITY



VARIANCE



$$\begin{array}{rcl} (x_1 - \bar{x})^2 & 10 & 100 \\ (x_2 - \bar{x})^2 & -30 & 900 \\ (x_3 - \bar{x})^2 & 20 & 400 \\ \hline & & 1400 \end{array}$$

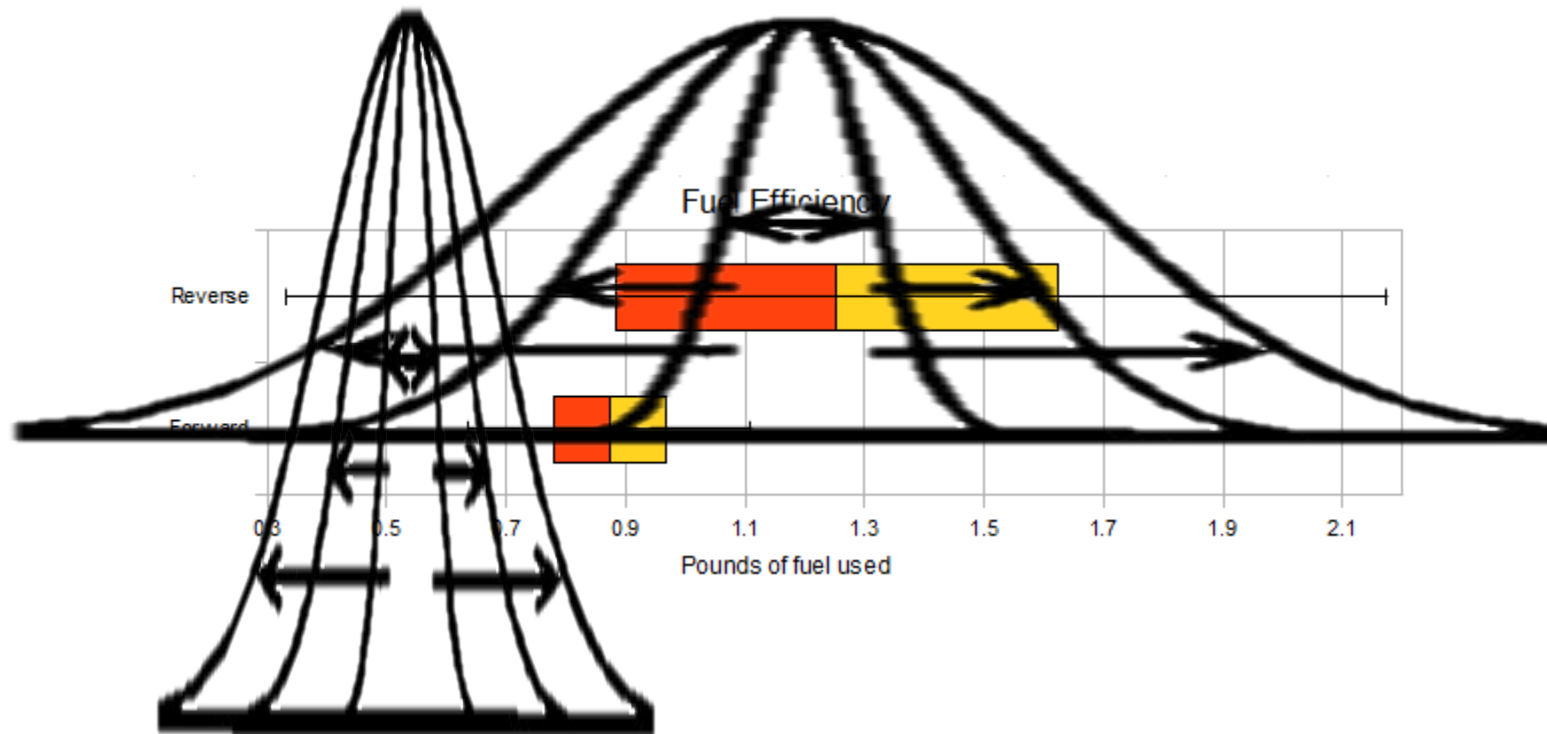


$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum x_i^2 - (\sum x_i)^2 / n}{n-1}$$

$$6^2 = \frac{\quad}{12}$$

$$\frac{466.67}{700} \quad n-1$$

VARIABILITY



PROBLEM # 2.8

Data set: 0, 7, 12, 5, 33, 14, 8, 0, 9, 22

c. Calculate the standard deviation

SAMIE LY
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STANDARD DEVIATION



is measured in the SAME UNITS as the data,
therefore easier to interpret.

Population

$$\sigma = \sqrt{\sigma^2}$$

Sample

$$s = \sqrt{s^2}$$

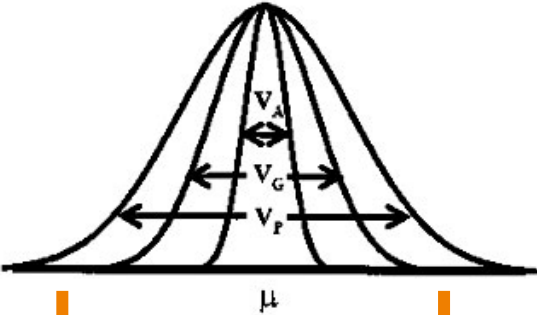
How large is the
Standard Deviation
from the mean.

Coefficient of Variation:

$$\left(\frac{\text{Standard Deviation}}{\text{Mean}} \times 100 \right) \%$$

Handwritten red annotations: 100% above the fraction and 80% below it.

VARIANCE



$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum x_i^2 - (\sum x_i)^2 / n}{n-1}$$

PROBLEM # 2.8

Data set: 0, 7, 12, 5, 33, 14, 8, 0, 9, 22

c. Calculate the standard deviation

SAMIE LY
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PROBLEM # 2.9

Answer the following questions about variability of data sets:

- a. What is the primary disadvantage of using the range to compare the variability of data sets?
- b. Can the variance of a data set ever be negative? Explain.
- c. Can the variance ever be smaller than the standard deviation? Explain.

PROBLEM # 2.9

BOTH
RANGE of 14



- a. What is the primary disadvantage of using the range to compare the variability of data sets?

The two data sets can

- have the same range
- be vastly different with respect to the data variation.

The range is greatly affected by extreme values.

BOTH
Sample size:
n=9

	1	1
	2	2
	2	4
	4	11
	5	12
	6	13
	7	14
	14	14
	15	15
Variance	25.94	31.28
Std Dev	5.09	5.59

Greatly affected
by EXTREME
numbers



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PROBLEM # 2.9

b. Can the variance of a data set ever be negative? Explain.

The variance can never be negative due to the squared deviations from the mean.

The square of any number, positive or negative will become positive.

The variance is always positive

PROBLEM # 2.9

c. Can the variance ever be smaller than the standard deviation? Explain.

The variance is usually greater than standard deviation.

HOWEVER, if the data are between 0 and 1, the variance will be smaller than the standard deviation.

Try this!

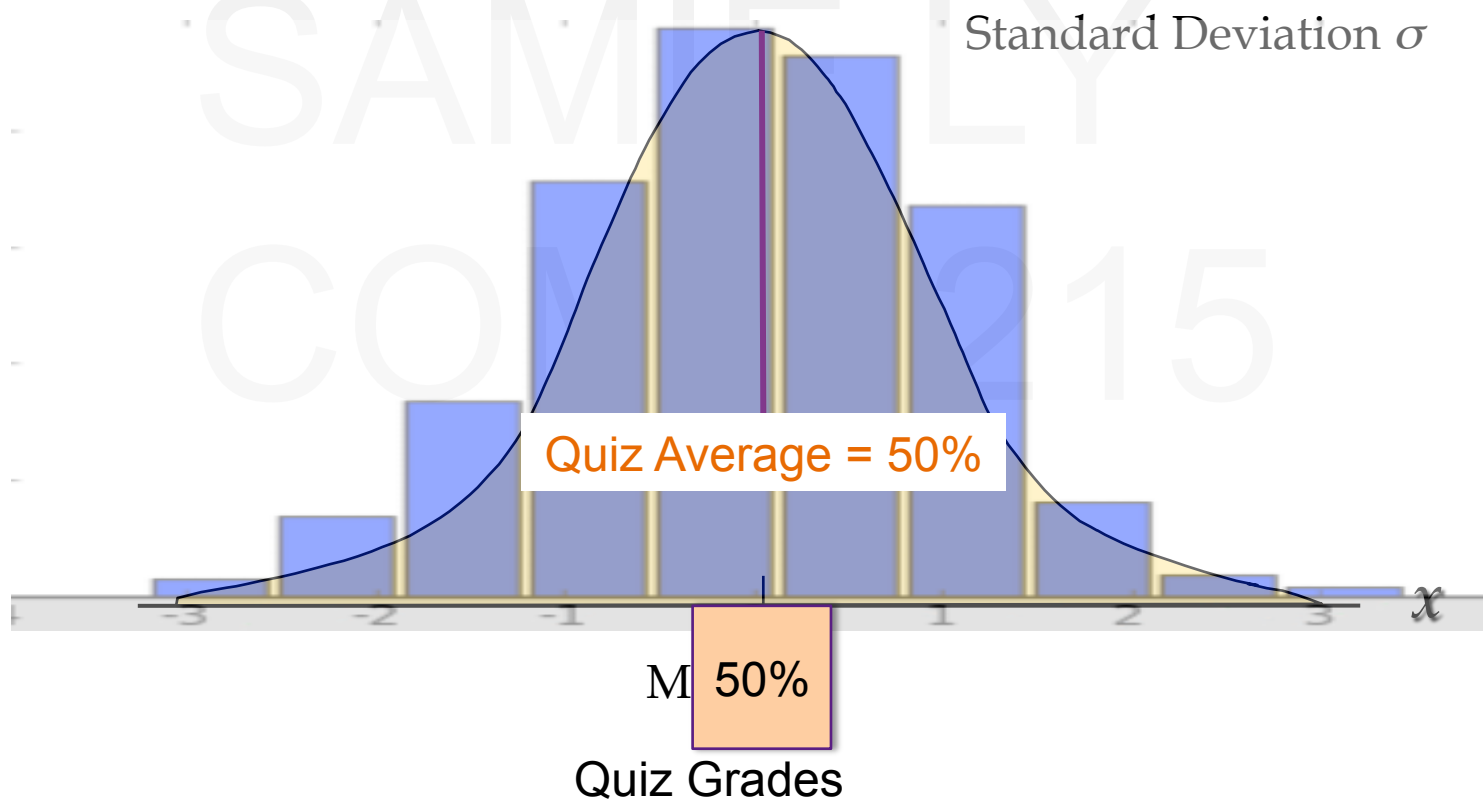
$s^2 = 4$, what is the standard deviation?

$s^2 = 0.8$, what is the standard deviation?



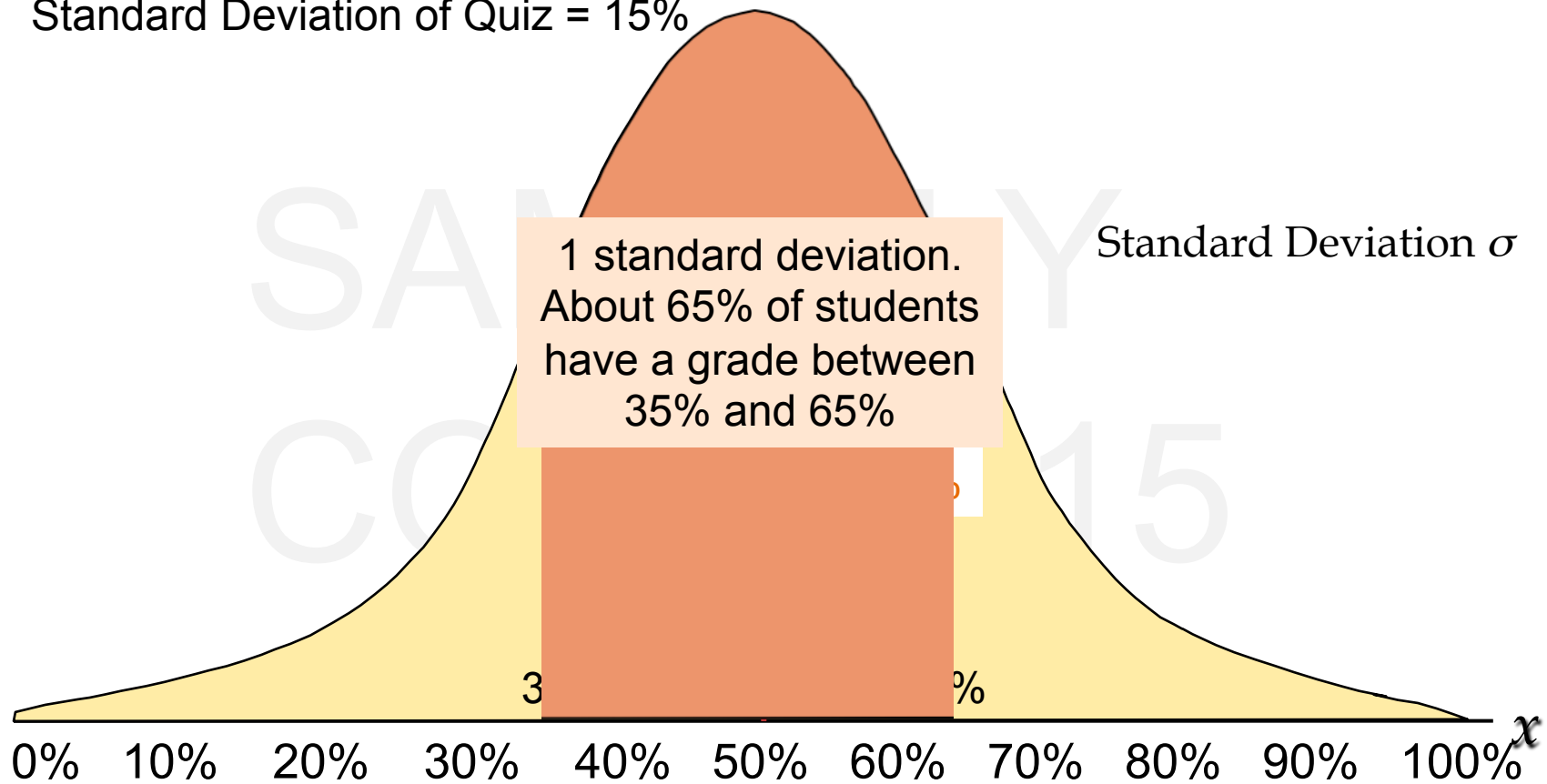
ESTIMATING DATA

- EMPIRICAL RULE
- CHEBYCHEV'S THEOREM



ESTIMATING DATA

Standard Deviation of Quiz = 15%



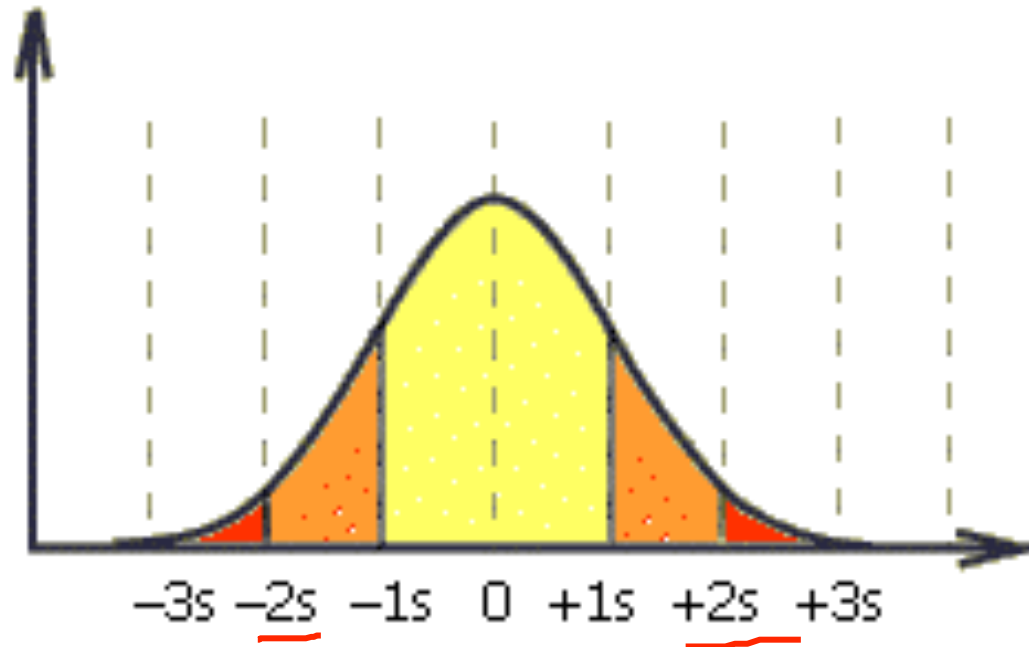
Quiz Grades

THE EMPIRICAL RULE

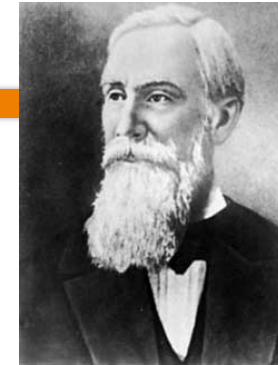
Bell-Shaped
Data!

Approximately ____% of all observations fall
Within ____ standard deviation of the mean.

Percent(%) of observations	Standard deviation

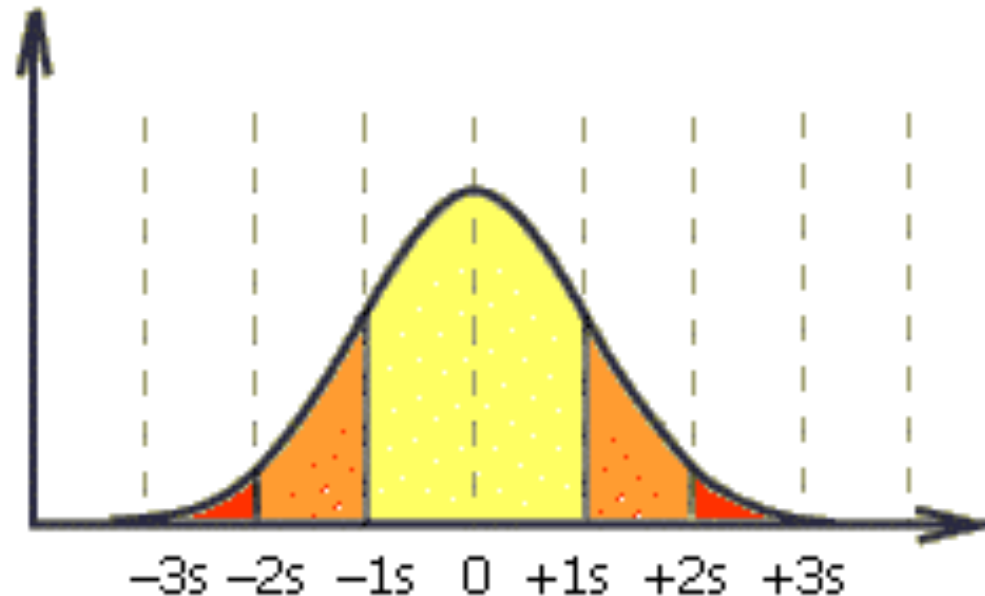


CHEBYCHEV'S RULE



**AT LEAST ____% of all observations fall
Within ____ standard deviation of the mean.**

Percent(%) of observations	Standard deviation



PROBLEM # 2.11

The annual salaries of the employees of a chain of computer stores produced a positively skewed histogram. The mean and standard deviation are \$ 28,000 and \$3,000 respectively. What can you say about the salaries at this chain?

NOTE: The histogram is not bell shaped, therefore you cannot use Empirical Rule. You must use Chebychev's theorem instead.

PROBLEM # 2.11

At least 75% of the salaries lie WITHIN 2 STANDARD DEVIATION

$$28,000 - 2(3,000) = 22,000\$$$

$$28,000 + 2(3,000) = 34,000\$$$

At least 88.9% of the salaries lie WITHIN 3 STANDARD DEVIATION

$$28,000 - 3(3,000) = 19,000\$$$

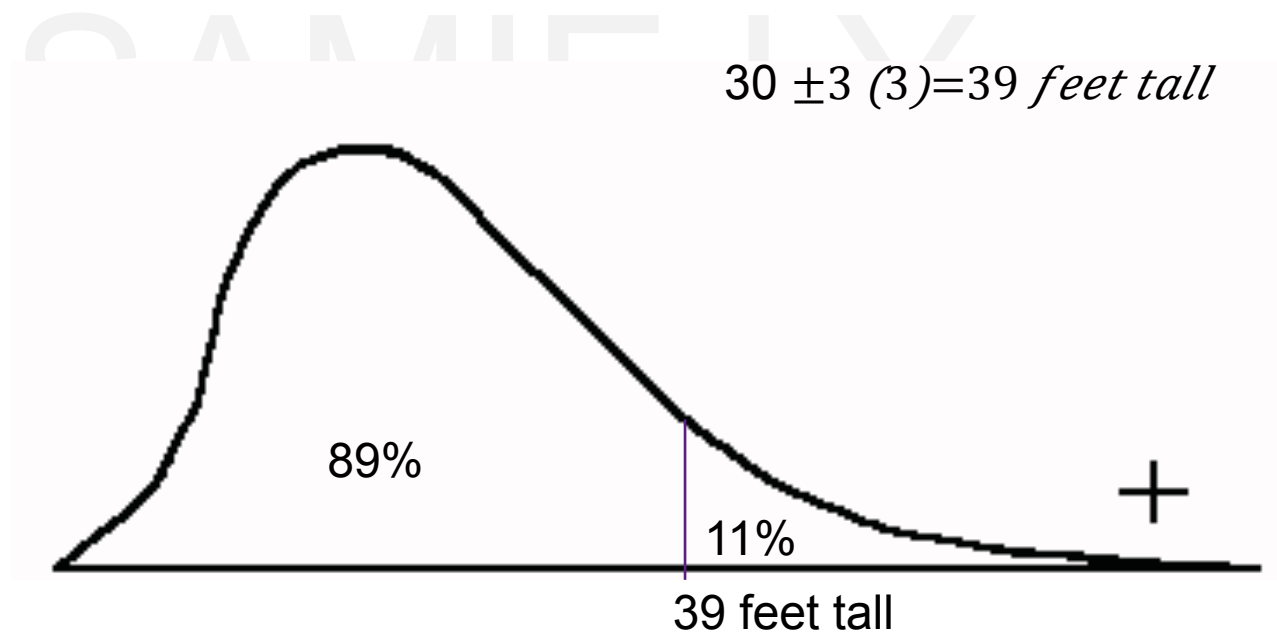
$$28,000 + 3(3,000) = 37,000\$$$

PROBLEM # 2.12

Land purchase decision. A buyer for a lumber company must decide whether to buy a piece of land containing 5,000 pine trees. If 1,000 of the trees are at least 40 feet tall, the buyer will purchase the land; otherwise, he won't. The owner of the land reports that the height of the trees has a mean of 30 feet and a standard deviation of 3 feet. Based on this information, what is the buyer's decision?

PROBLEM # 2.12

Using Chebychev's rule, at 3 standard deviation



Will not buy because it is not 20% = 1000/5000

OUTLIERS USING Z-SCORE



Z-score = 1, at 1 standard deviation from the mean

Z-score = 2, at 2 standard deviation from the mean

Z-score = 3, at 3 standard deviation from the mean

$$Z = \frac{x - \bar{x}}{s}$$

Any value outside of 3 standard deviation is considered an outlier.

● Percentiles, Quartiles, and Box and Whiskers.

Midterm Preparation



PERCENTILES, QUARTILES, AND BOX AND WHISKERS

- PERCENTILE
- INTERQUARTILE RANGE (IQR)
- BOX PLOTS
- FIVE NUMBER SUMMARY

PERCENTILES



Arrange the dataset by ascending order (smallest-largest)

Compute i

$$i = (p/100)n$$

- (a) If it is not an integer, ROUND UP
- (b) If it is an integer, the p th percentile is the average of the values in position i and $i + 1$

QUARTILES are 25th, 50th, 75th percentiles.

PROBLEM # 2.8

Data set: 0, 7, 12, 5 , 33, 14, 8 , 0 , 9 , 22

- a. Calculate the 50th, 25th percentiles of the data above.
- b. Find the interquartile range

PROBLEM # 2.8

$$i = (p/100)n$$

Data set: 0, 7, 12, 5, 33, 14, 8, 0, 9, 22

a. Calculate the 50th, 25th percentiles of the data above.

Order Data set: 0, 0, 5, 7, 8, 9, 12, 14, 22, 33.

$$i = (50/100) * 10 = 5$$

$$= \frac{(5^{\text{th}} \text{ value} + 6^{\text{th}} \text{ value})}{2}$$

$$= \frac{(8+9)}{2} = 8.5$$

The 50th percentile value = 8.5

$$i = (25/100) * 10 = 2.5 \text{ Round up!}$$

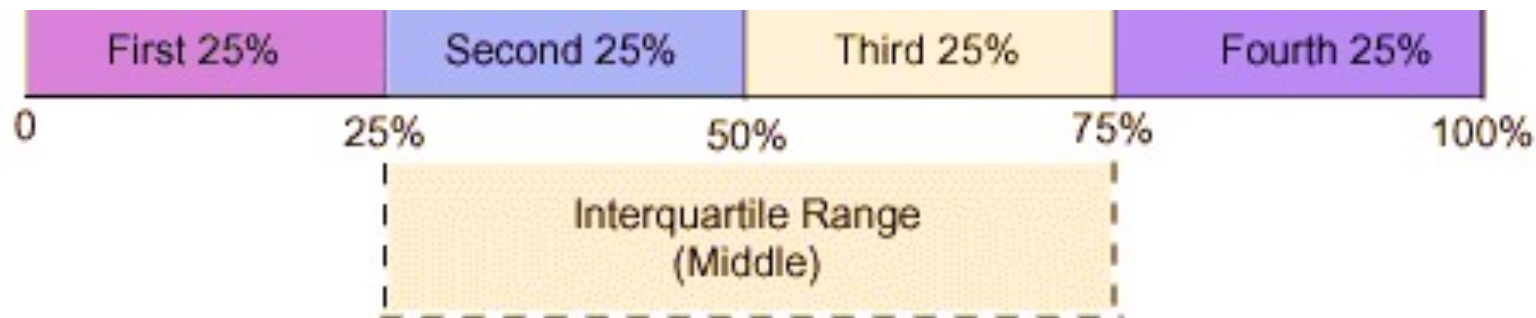
$$= \underline{3^{\text{rd}} \text{ value}}$$

The 25th percentile value = 5

INTERQUARTILE RANGE (IQR)



$$Q3 \text{ (75}^{\text{th}} \text{ percentile)} - Q1 \text{ (25}^{\text{th}} \text{ percentile)} = \text{IQR}$$



The largest number – smallest number in your sample is considered the RANGE. Example: 20-2 = 18.

PROBLEM # 2.8

$$i = (p/100)n$$

Data set: 0, 7, 12, 5, 33, 14, 8, 0, 9, 22

b. Find the interquartile range

Order Data set: 0, 0, 5, 7, 8, 9, 12, 14, 22, 33.

$$IQR = 9$$

$$i = (75/100) * 10 = 7.5 \text{ Round up!}$$

= 8th value

The 75th percentile value = 14

BOX PLOT & WHISKERS



Smallest observation

Largest observation

First Quartile (Q1)

Second Quartile (Q2)

Third Quartile (Q3)

Interquartile Range (IQR)

Upper Fences, Lower Fences (Inner and Outer)

Whiskers

Outliers

BOX PLOT



Upper Inner Fence, Lower Inner Fence

- $1.5(IQR)$ below $Q1$
- $1.5(IQR)$ above $Q3$

Upper Outer Fence, Lower Outer Fence

- $3(IQR)$ below $Q1$
- $3(IQR)$ above $Q3$

Whiskers

- Smallest and Largest values **WITHIN THE LIMITS**

Outliers

For Box Plot: Outside the Limits

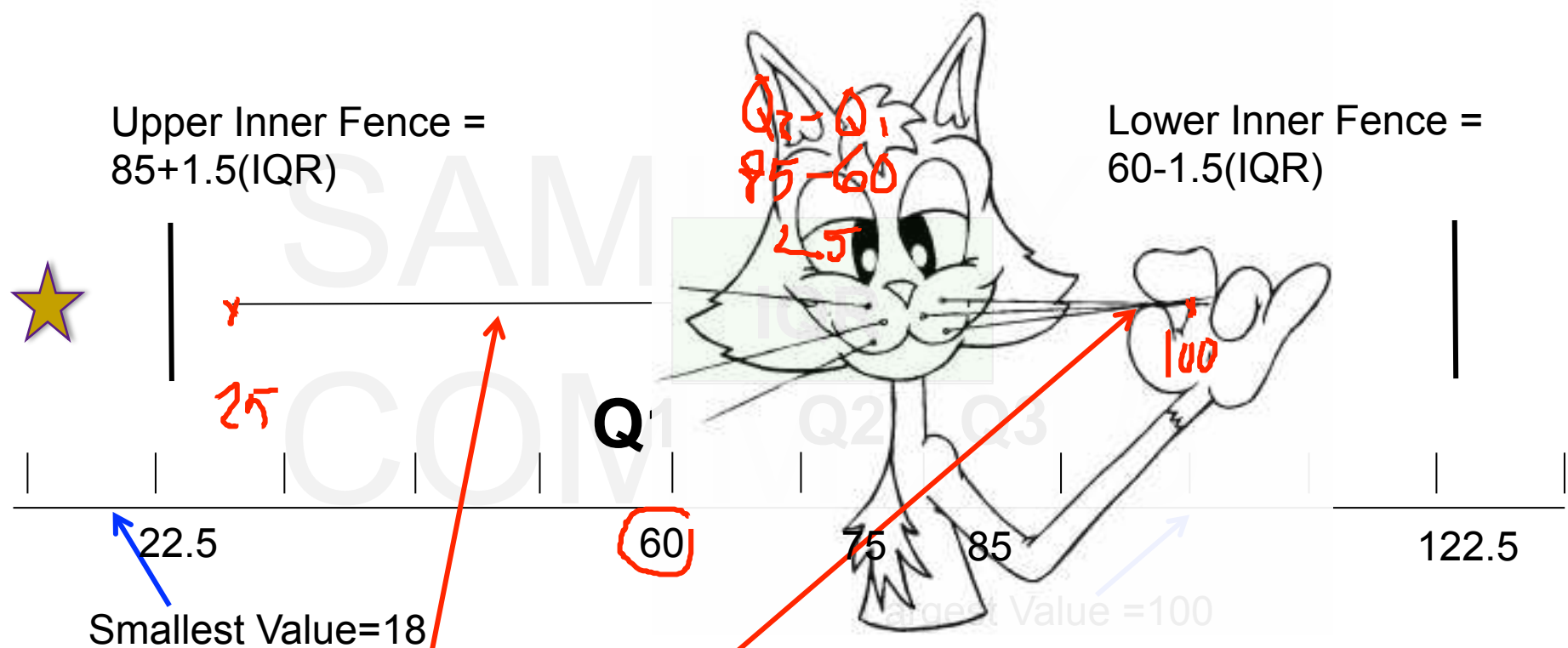
For Empirical/Chebyshev's Rule: Outside the 3rd Std Dev

PROBLEM # 2.13

Suppose a data set consisting of exam scores has a lower quartile $Q_L = 60$, a median $m = 75$, and an upper quartile $Q_U = 85$.

The scores on the exam range from 18 to 100. Without having the actual scores available to you, construct as much of a box plot as possible.

Partial Data: 18, 25....100



Whiskers



OUTLIERS

FIVE NUMBER SUMMARY



Smallest observation

Largest observation

First Quartile (Q1)

Second Quartile (Q2)

Third Quartile (Q3)

Interquartile Range (IQR)

Upper Fences, Lower Fences (Inner and Outer)

Whiskers

Outliers

Midterm Preparation

THE CONTEXT



One of the major measures of the quality of service provided by any organization is _____.

The **speed**

with which it responds to customer complaints.

The data below give

the number of **waiting days**

Between the receipt and resolution of a complaint



HOX PLOT PROBLEM

A cellphone software company was receiving complaints about the installation of cellphone software.

The experimental unit to the data is: Waiting Days

72	20	29	23	30	27	25	32	4	29
29	31	27	60	13	51	36	5	26	1
12	22	14	28	35	26	9	34	3	1

HOX PLOT PROBLEM

72	20	29	23	30	27	25	32	4	29
29	31	27	60	13	51	36	5	26	1
12	22	14	28	35	26	9	34	3	1

Construct a box plot for the data set and comment on the nature of the plot including identifying outliers, if any.

HOX PLOT PROBLEM

72	20	29	23	30	27	25	32	4	29
29	31	27	60	13	51	36	5	26	1
12	22	14	28	35	26	9	34	3	1

Compute the proportion of observations that are within one, two and three standard deviations of the mean. Does the empirical rule apply for this data set? Identify outliers, if any.

HOX PLOT PROBLEM

72	20	29	23	30	27	25	32	4	29
29	31	27	60	13	51	36	5	26	1
12	22	14	28	35	26	9	34	3	1

To improve services, management wants to grant a special offer to customers whose complaints take unnecessarily longer periods to resolve. However, it is desirable that no more than 25% of the customers who lodge a complaint would receive the special offer. Approximately, what number of waiting days between the receipt and resolution of a complaint may be used for the cutoff?

PREVIEW ON LESSON 3

SAMIE LY
COMM 215

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